ELEMENTARY LOGIC

WITH SPECIAL APPLICATION TO METHODS OF TEACHING

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PREFACE

This book is designed primarily for students in normal and training schools. Throughout its pages the bearings of logic upon the problems of education have been constantly kept in mind. The relation between certain phases of logic and the teacher's class-room work has been emphasized.

While the interests of the future teacher have received especial attention, the subject-matter is so arranged that the pedagogical applications may be ignored, thus adapting the book for use in colleges. When used in this way, all of Part VI may be omitted.

The writer believes that a text-book in logic should have two aims: first, to preserve the fundamental principles of formal logic; second, to bring the logical doctrine into harmony with the recent developments of functional psychology. The psychological creed adhered to throughout the book is frankly genetic, and the developmental conception is in evidence in the treatment of topics, notably Judgment, the Functional Value of Reasoning, and the Fallacies.

The traditional connections between logic and language have been reiterated, in the belief that they are fundamental and that they furnish a valuable apperceptive basis for the introduction of the subject to students fresh from the study of languages. The effort

has been made, however, to bring the linguistic side of logic into close accord with the underlying psychological processes.

Exercises have been arranged with the aim of stimulating the student's activity in the invention of examples illustrating the various phases of logic. The value that comes to the student from the exercise of his own inventive ingenuity cannot be doubted. The usual type of exercises is predominantly analytical. Constructive work aids materially in developing a full comprehension of the subject. The student who is able to make his own examples puts his knowledge to the test of a practical application.

In addition to the exercises, a set of questions on the text of each chapter is provided. These questions are designed as an aid to both teacher and student in focussing attention upon the most important points in the text.

Both exercises and questions are placed after the chapters in the belief that this arrangement will prove more convenient than the usual one of grouping them at the end of the book.

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CHAPTER I.—INTRODUCTION

1. Definition of Logic.—Logic may be defined as *the science of the laws of thought and the art of using them.* This definition serves as an indication of the nature of logic, though it must be regarded as merely preliminary and tentative. Our whole course is to show what ground logic covers, and also how its laws are to be made of practical use.

We must pause to consider the difference between science and art. The former teaches us to know, the latter to apply our knowledge to concrete situations. Science gives us abstract statements of laws, while the corresponding art shows how these abstract formulas are to be made applicable to practical needs. Thus the science of psychology includes statements of laws governing the accumulation and organization of knowledge, the art of teaching puts these scientific principles to use in school-room practice. The science of logic is sometimes called *Pure Logic,* the art of logic *Applied Logic.* In this book no hard-and-fast distinction will be made between the two aspects of the subject, though the discussion of laws will emphasize the "pure" side, the practice of the various exercises the "applied" side.

A moment's explanation is needed to dispose of the word "thought" as employed in the above definition. Thought, as used in this connection, means any effort to determine the meaning or implication of a present form of experience. Those parts of our experience that hold our attention are worked up into ideas or thoughts which prompt our reasoning powers to bring them into relation with other ideas or thoughts. This attempt to organize our ideas is the work of intellect or reason. In order to prevent intellect's going astray in the heat of enthusiasm to extend our knowledge beyond our present, momentary experiences, we have to see to it that the procedure follows certain principles of valid reasoning set forth in logic.

Sigwart, the German logician, indicates the need and function of logic very clearly in the following passage:

> *If we consider the nature of our thought, we find that an important part of it is engaged in the attempt to arrive at propositions which are certain and universally valid, but that it frequently fails to do this when left to its natural development. Hence arises the problem of ascertaining the conditions under which this object can be attained, and of determining in accordance with those conditions the rules to be followed in its attainment. The solution of this problem would place us in possession of a technical science of thought, directing us how to arrive at certain and universally valid propositions. Such a science we call Logic.*[1]

2. Relation to Other Sciences.—Logic is closely related to and in a measure dependent upon the sciences of grammar, rhetoric, and psychology. Logic is concerned with the *meanings* of our various thoughts. We

[1] Sigwart, *Logic* (Eng. tr.), Vol. I, p. 1.

never think without intending or meaning something that points beyond the thought considered as a mere mental process. These meanings are useful to us as guides to various kinds of action. If I go through the psychological process of noticing that snow is on the ground, if I summon up from past experience the interpreting thought that snow is cold, if I conclude that consequently it will be worth while to wrap up well before venturing out, it is clear that I reach this conclusion only in view of contemplated action. But I expect such a conclusion to guide not only my own action, but also the actions of other members of my family. In other words, though I may not in the first instance give oral expression to the conclusion that I must wrap up well before going out, I yet feel that when I choose to express it in language it will gain the assent of others. The thought has meaning for my action just in so far as, if expressed, it would also have a compelling force in controlling the actions of others. This meaning-value of a reasoning process is what we intend when we say that it is true.

Before my meaning can be put to the test of assent by others, it must have a language form. This language form must accomplish two results: first, it must state exactly what I think, so as to defy my own criticism; second, it must stimulate in other minds essentially the same thought that I seek to convey. Both grammar and rhetoric lay down principles governing the accurate statement of thought, and from these principles logic must borrow when applying the test for meaning. Grammar teaches the forms and syntactical relations of

words. Forms and relations express modifications and shades of meaning. Logic assumes grammatical science in so far as such science is the basis of meaning. Rhetoric is in part an advanced grammar, in part a science of style. Those principles of rhetoric which are concerned with rendering the meaning clear are adopted as a basis of logic. But that part of rhetoric which treats of style has no logical value; for logic is continually engaged in stripping off the outer adornments of style to get at the framework of meaning, which they half reveal and half conceal.

Logic is an expression of certain mental processes. These are analyzed, described, and explained in psychology. Hence logic must resort to psychology for these analyses, descriptions, and explanations. The difference between psychology and logic is that the one is interested in the intellectual processes merely as such, while the other is interested in their results.

The obligation which logic owes to certain sciences is more than repaid by the fact that it contributes to all sciences their mode of procedure. Every kind of scientific exposition, be it book or recitation, lecture or experiment, must be logical to be fruitful. Logic thus offers the key to success or failure in method, and according as scientific writings succeed or fail are they examples of good or bad logic.

3. Nature of the Laws of Thought.—The word law is used in so many ways that we need to consider its meaning in the phrase, *laws of thought.* By law we usually mean either an artificially devised enactment under the control of human will, or else a uniform mode

of nature's operation, which man discovers and states, but cannot control. An example of the one are the laws passed by the Fiftieth Congress; of the other, the law of falling bodies. Laws of thought are in the class of natural laws. They were discovered and expressed in language by man, but cannot be made and unmade at his bidding. Our acts may bring natural laws into conflict—with a usual consequence of disaster to oneself; as when a man bent on suicide brings about a situation where the laws of his physical well-being conflict with those of falling bodies by projecting himself from a third-story window to the pavement below. One can never annul them as can the legislator the man-made laws on our statute-books. The natural laws governing our logical thinking are sometimes called regulative, because the strict observance of them forestalls wrong conclusions. We often find that they conflict with certain other natural laws governing our mental behavior. Thus there is a general psychological principle to the effect that an unusual stimulus in the way of sudden shock produces very strong emotions of joy, fear, or anger, as the case may be. A mind which is in this state is prone to act without due deliberation, without regard to the laws of logic, and hence to arrive at insane conclusions. Here is an instance of conflict between these various natural laws within the psychological sphere. In a word, it may be said that natural laws of thought are regulative, like those of health. When discovered and obeyed, our thinking goes well; when either wittingly or unwittingly ignored, our thinking gets into the most inextricable tangles.

4. Importance of Logic.—Many persons on beginning the study of logic are at a loss to see its practical value. They argue that they have previously arrived at correct conclusions without it, and they fail to see why they should not be allowed to continue in the same happy-go-lucky way. The same line of argument would be equally valid against medicine and all other regulative sciences. People live on for years without feeling the need of the doctor. But let them once fall seriously ill and they see that the science of medicine is not all in vain. The man whose affairs go well may live at peace with all men and see little good in the profession of law. But let him suffer financial reverses and the lawyer's aid may be the only recourse that will unravel his knotty problems. Arguing in the same way, the grammar and the dictionary need never be consulted. The unlettered man succeeds in expressing his meaning in a fashion that is sufficient unto his needs. Why then this eternal thumbing of the dictionary to learn the correct use and pronunciation of words? Why this tiresome study of grammar to get at principles of syntax? The answer comes readily to mind. Because there is a standard of correctness in speech and thought that we are better off for having attained, and only by the study of the appropriate science and art may we hope to approach the standard. So in logic there are correct standards or forms of thinking which are a measure of reasoning processes. Apply them, and we arrive at the truth; fall short of them, and we land in error. Now error would not be so mischievous were it not that our conclusions condition our actions. But

since all reasoning is in view of action, and since action is the only means within our power for promoting our physical and spiritual welfare, it becomes at once apparent that the direst of consequences may flow from negligence of logical laws.

5. Mental Processes Presupposed in Logic.—It was stated above that logic refers to psychology for an analysis and description of such mental processes as are presupposed in logical operations. The first of these is called *sense-perception.* It gives us the material of thought. Through this function of the mind we get a knowledge of a world of separate, individual things. Sense-perception is dependent upon the several senses of sight, hearing, touch, temperature sense, muscular sense, smell, taste, and the so-called organic sensations which come mainly from the viscera. Some or most of these senses are used whenever one perceives any material object, such as the desk where he sits, the book he holds in his hand, etc. The second process is called *conception.* It is the mental activity which compares the things given by sense-perception, noting wherein they are like and unlike. It forms a concept or class-notion (also called a general notion or idea) of those things which have noteworthy characteristics in common. Such a class-notion is illustrated by our idea of ruminant animals as those which chew their cud. The third process is *judging.* This is the mental function of connecting concepts. When expressed in language-form, we name its product the judgment. This consists of two terms which stand as symbols of concepts united by the copulative verb, *to be,* in an appropriate

form, or by some other verb which implicitly contains it. *The day is pleasant,* is the result of a process of judging expressed in the language-form of the judgment. The fourth process is called *reasoning.* This is the mental act of uniting two operations of judging in such wise as to suggest a third one, which is not identical with either of those united, but conveys a truth which necessarily flows from the two when brought into comparison. The logical vehicle of reasoning is the syllogism or other language-form closely related to it. It is an example of reasoning when we infer that it has been raining because the pavement is wet. Expressed in the full logical form, such reasoning is as follows:

Wet pavements are due to rain;
This is a wet pavement;
Therefore this condition is due to rain.

There is another phase of mental procedure that needs to be noted here—one, indeed, that is implied wherever the mind operates to produce logical products, viz., *apperception.* Apperception is the process of relating a present fact to some past experience which explains it as its class, ground, cause, or reason. This relating function of the mind must not be construed as something unique in character and special in mode of operation. We find it everywhere in the intellectual processes—in sense-perception, conception, judging, and reasoning. Without it our separate sensations would not be moulded together into the individual things that constitute our sense world; the several things would not be grouped together into a class or kind; our class-

notions or concepts would not take the judgment form; nor would our judgments unite in the final logical procedure of reasoning. Whenever a present experience engages our attention it forthwith challenges the mind's relating activity to build it on to any phase of previous experience that is discovered to have some bearing upon it. It is possible that such previous experience is merely mechanically related to it in space or time. Under such circumstances the present fact drags up the past, interpreting experience owing to the working of the principle of association by contiguity. This is illustrated when a given air calls up the face of a friend who was with us when, on a former occasion, we heard the same tune played. Here the relation is trivial and superficial; nevertheless the air gets greater meaning for us as a present experience, by reason of the reestablishment of this past time-relation. More often the previous experience which comes to the aid of the present thought comes forward because of a profound likeness between them. This is association by similarity. The principle of association may be sufficient in this instance to account for the presence of the escort of old experience, but we must go deeper into the mind's workings to explain the apperceptive process. Once the likeness between the new and the old is detected, the mind holds on to the relationship, examines it with great minuteness, detects in the midst of similarity elements of dissimilarity, denies the relationship in so far as unlikeness appears, but strengthens its affirmation in respect to the elements of similarity, often going so far as to assert identity within the prescribed limits. This

is the higher and more intellectual aspect of apperception—the phase of it that underlies all logical reasoning.

6. THE TEACHER'S INTEREST IN LOGIC.—Logic is of especial importance to the teacher. For it shows him the form that the data of experience must assume in order to be quickly and certainly united. It also enables him to avoid errors to which we are liable in relating our present fact to its apperceiving basis of past experience. For these reasons good method of instruction is quite as much a question of logic as of psychology. Indeed, a careful analysis of the "formal steps" about which we hear so much in books on methods of teaching shows them to be attempts to solve problems in the logic rather than the psychology of instruction.

Aside from this contribution of logic to teaching, we have already seen on page 4 how every attempt to teach either by book, experiment, or oral instruction hinges upon the question of what the logic of the situation demands.

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REVIEW QUESTIONS

1. *Cite some reasons for and against defining a subject at the beginning of its study.*
2. *Distinguish between a science and an art.*
3. *What do you understand by the term "thought" as used in the definition of logic?*
4. *Show how we connect the new experience with the old in the organization of thought.*
5. *What does logic get from grammar? from rhetoric? from psychology?*
6. *How do you understand the contention that every thought has a "meaning," and that every "meaning" ends in action?*
7. *How is logic useful to the other sciences?*
8. *Distinguish between the two usual meanings of the term "law."*
9. *What do you understand by a regulative science?*
10. *Name the mental activities involved in the logical operations of the mind.*
11. *What do you understand by apperception?*
12. *What do you understand by "association by contiguity"? by "association by similarity"?*
13. *How is logic useful to the teacher?*

EXERCISES ON CHAPTER I

1. Name the science and the art aspects of five branches of study you have already pursued.
2. Cite and explain three instances in your recent personal experience where old forms of experience have come to mind in connection with a present experience.
3. Distinguish in any two instances you choose to name between your thought and the meaning of your thought.
4. Cite an instance in your personal experience where a thought's meaning led to action.
5. Cite an instance from your own experience in which you made certain of the truth of your idea by submitting it to the assent of others.
6. Name two statute laws and two natural laws.

7. Name two other regulative sciences besides those mentioned in the text.

8. Exemplify sense-perception by sight, hearing, sight and hearing, touch, temperature sense, muscular sense.

9. Make an illustration of conception, judging, reasoning.

10. Cite an instance from your own recent experience illustrating apperception.

11. Illustrate association by contiguity; by similarity.

12. Illustrate concretely how logic may be useful in each of the ways mentioned in the development of a lesson (make free use of your imagination here).

PART I.—TERMS

CHAPTER II.—KINDS OF TERMS

7. Definition.—A term is *any word or word-group that is used as a symbol for an object or class of objects of thought.* The object of thought for which the term stands as a sign may be either material or immaterial; either a thing or a quality. Examples: *dog, blue, this man, Christopher Columbus, the book which you were reading, the problems that the teacher assigned to her pupils for home study, to-morrow's logic lesson, incommensurability, responsible, sameness.*

8. How Terms are Composed.—The varied list of examples just given should make it clear that terms are language-forms which stand as *signs for ideas.* Our ideas refer to realities that are either simpler or more complex. If relatively simple, the term which stands for the idea of the thing is usually one word, or at most two. But if the thing referred to by the term is complex, then it frequently happens that the term becomes descriptive, so that its noun part is qualified or restricted by adjectives, phrases, or even subordinate clauses. It not infrequently happens, however, that even in the latter case the idea of the whole predominates over the idea of the complex parts; under such circumstances the term is in the course of usage pared

down to one or two words. As an example of a term designating a simple idea may be mentioned the word *star;* of one referring to a complex idea, *the subway system of New York;* of one referring to an idea in which the thought of the whole prevails over that of the parts, *Brooklyn Bridge.*

Above all it should be remembered that terms have a representative function, serving to refer both to things material and immaterial, and to our ideas of them also.

9. Words and Terms.—The beginner in Logic, especially if he is fresh from the study of English, is quite prone to confuse words with terms, thinking mistakenly that every word may serve by itself as a term. While it is true that terms consist either of words or groups of words, it by no means follows that every word can serve as a term. The only parts of speech which can discharge this function are nouns, pronouns, adjectives, and the predicative part of verbs. Hence these parts of speech are said to include all *categorematic* (asserting) words. All the other parts of speech, viz., adverbs, prepositions, and conjunctions, consist of *syncategorematic* words, *i. e.*, words that never stand alone as terms, but merely play the subordinate rôle of helping in the formation of terms consisting of several words. It should be remembered that though categorematic words may be used alone as terms, they often combine with other words to constitute complex terms. We may cite the following examples of categorematic words: *horse, William, red, she, playing* (as used in the sentence, *John is playing, i. e., plays*). Examples of syncategorematic words are: *for, and, bravely.* The following

terms show both categorematic and syncategorematic words combined to symbolize ideas representing complex realities: *the shade of the tree, the cattle on a thousand hills.*

10. Terms Considered from the Grammatical Stand-point.—Defined from the view-point of grammar, a term is a *categorematic word, including any modifiers it may have, whether adjectives, adjective phrases, or adjective clauses.* Examples: *the blue sky, Mary, friend, the man in the boat, the friend with whom you were walking evening before last.*

What the grammarian calls a declaratory or conditional sentence the logician calls a proposition. For purposes of logic every sentence such as the grammarian calls a principle sentence may be analyzed into two terms and a copula—the *first* or *subject-term,* the *second* or *predicate-term,* and connecting them the form of the present tense of the copulative verb, *to be,* demanded by the rules of agreement. All modifiers of the subject belong to the first or subject-term; all modifiers of the predicate to the second or predicate-term. Example: *The lady in the carriage with whom I saw you conversing just now is the sister-in-law of an old friend of mine who was in college with me.* In this involved complex sentence the first term is, *the=lady=in=the=carriage=with=whom=I=saw=you=conversing=just=now;* the copula is the verb *is;* the second term is, *the=sister-in-law=of=an=old=friend=of=mine=who=was=in=college=with=me.* The two terms are hyphenated with the double hypen to show that all the words belong together as parts of one term.

11. Classification of Terms.—Terms may be classified in several ways, in accordance with the purposes held in view. The following table shows the most important classes:

I. Nouns (including also words and word-groups so used) are classified according to their numerical implication into	(*a*) Singular (mostly proper nouns). (*b*) General (common) (*c*) Collective.[1]	
II. Terms are classified as to whether they refer to a thing, and a quality inhering in a thing; or a quality thought of apart from a thing into	(*a*) Concrete. (*b*) Abstract.	
III. Terms are paired according to whether they affirm or negate a quality into	(*a*) Positive (*b*) Negative	Contradictories.

12. Definitions.—A singular term is a term *which is used to apply to only one object;* such as, *John Jones, Westminster Abbey, the house that Jack built, Socrates, this dog, my hat.*

Singular terms may be subdivided into *individual* and *collective.* The former designate single objects that cannot be analyzed into smaller units without destroying their character; the latter name single aggregates of interchangeable units. Examples: singular individual, *John Jones, the Lusitania;* singular collective, *the New York Police Force, the 57th General Assembly of the State of New York.*

A general term is a *class name.* It is employed to designate the class as a whole, the class-notion, or any

[1] Collective terms are either singular or general, depending upon whether the group referred to is *one* or *more than one.*

individual in the class; as, *dog, horse, soldicr, bridgc, member of Parliament.*

A general term is said to be used *distributively* because it is equally applicable to the class as a whole or to each individual constituting the class; thus the term *girl* is applicable to a school-room full of female pupils and also to each one of them, as Mary, Helen, Grace, Florence, etc. In distinction from this we have the terms that are used only *collectively,* being applicable to the group-whole, but not to the individuals composing it; thus the term *class* would apply to the above pupil-group but not to any individual of the group.

A collective term is *one which points out a group composed of distinct individuals, but where the group idea is more prominent than the individuals composing the group;* as (singular collectives) *the Tenth New York Volunteers, the American army, my regiment;* (general collectives) *church, regiment, navy, flock of geese.*

A concrete term names *a thing, or a quality conceived as inhering in a thing;* as, *red apple, lamp, Mr. Smith, invincible.*

An abstract term names *a quality conceived as an object of thought apart from the thing in which such quality inheres;* as, *redness, goodness, fluidity, brilliancy, invincibility.*

It should be carefully noted that the distinction between a concrete and an abstract term hinges properly upon the mental process implied. In a general way the functions of sense-perception, memory, and imagination give us concrete experiences; those of conception and reflective thought furnish us with abstractions. Yet it not infrequently happens that the products of the latter functions, if often brought into the focus of attention, come in the course of time to have a reality as commanding

as the veriest sense experiences. Thus it is that the philosopher's and the poet's worlds are built up into a cosmos that is as real for them as any possible sensations. And in the lives of most of us ideals gain a foothold in consciousness that enables them to dominate our most important actions completely; for "the mind grows by what it feeds on." By a gradual process of a similar nature abstract ideas and the terms which denote them take on a certain concreteness of import that renders them difficult to distinguish from true concretes, the only sure test being whether or not sense experiences correspond to them. This tendency of the mind to treat abstract ideas (and the terms which symbolize them) as though concrete becomes a dangerous habit with persons of speculative and imaginative tendencies, leading to the fallacy of "hypostasizing abstractions." Examples of true abstract terms that have gained a partial concreteness for many minds are such words as *mind, memory, reflection, reason, reciprocity, democracy, nationality.* It is such words that occasion the chief difficulty in getting a satisfactory classification.

Contradictory terms *belong in pairs, one of which affirms and the other of which negates a given attribute,* the former being called the positive and the latter the negative term; as, *good* and *not-good, impressive* and *non-impressive, patient* and *non-patient.*

Contradictories must not be confused with terms which merely indicate degrees of opposition, such as *greater* and *less, large* and *small,* etc. True logical contradictories are best expressed by the awkward prefixes *not-* or (if prefixed to Latin derivatives) *non-*. The prefixes *in-* (*im-*) and *un-* have in practically all cases lost their original contradictory force through gradual linguistic changes, so that, for instance, *impatient* no longer means simply *non-patient.*

Another distinction that is often made is that between absolute and relative terms. *An absolute term is one naming a thing that has no necessary connection in thought with any other particular thing. A relative term is one that names an object that stands in necessary thought-connection with some other particular thing.*

Examples of the former are, *man, tree, mountain-chain;* of the latter, *shepherd, father, husband, king.* In the case of relative terms the name of the object necessarily related is often called *correlative;* such, for instance, are *flock, child, wife, subject,* in relation to the relative terms cited above.

The student must carefully distinguish contingent connections due to association of ideas from necessary relations due to the interdependent natures of the things named by relative terms. All things are connected in the mind through association, but the connections may be various; so that the mention of a name suggests a certain term to one mind and a different one to another. Thus the term *ship* may suggest *ocean* to one person, *port* to another, and *sail* to a third. Despite the intimacy of the association none of these terms are true correlatives to ship. A good test to apply is to think whether or not the *objects named by the terms suspected of correlation would cease to have the mode of existence implied if the supposed correlative could not be applied.* Thus a man would cease to be a parent were we to annul the relation expressed by child; but a ship would not cease to be a ship by annulling the connection expressed by port, sail, or ocean.

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REVIEW QUESTIONS

1. *What are the main points that the definition of "term" given above seeks to emphasize?*

2. *What do you understand by "an object of thought" as distinguished from a "thing"?*

3. *What is the function of terms?*

4. *How does the simplicity or complexity of the thing symbolized by a term influence the form of the term?*

5. *What two kinds of reference do terms have?*

6. *How may the beginner confuse words and terms?*

7. *What parts of speech are categorematic? syncategorematic?*

8. *How would the grammarian define a term?*

9. *What part of a sentence constitutes the first term? the second term?*

10. *What do you understand by the classification of nouns according to their "numerical implication"?*

11. *Show how collective terms may be either singular or general.*

12. *What three functions as symbols do general terms perform?*

13. *Where is the focus of attention in case of an object symbolized by a collective term?*

14. *How do you understand the expression, "a quality conceived as inhering in a thing"?*

15. *Upon what does the psychological distinction between the concrete and the abstract depend?*

16. *What mental functions give us concrete experiences?*

17. *Upon what mental activities does abstraction depend?*

18. *Show how the products of abstraction come to be regarded as concrete and "real."*

19. *To what fallacy of thought are persons of idealistic and speculative tendencies pronc?*

20. *Explain the phrase, "hypostasis of abstractions."*

21. *What confusion is liable to arise in case of contradictory terms?*

22. *How is such confusion to be avoided?*

23. *What prefixes is it best to employ as signs of true negative terms?*

24. *What do we mean by relatives? correlatives?*

25. *Distinguish between relatives and terms connected by association.*

EXERCISES ON CHAPTER II

1. Make five examples of terms of different degrees of complexity.

2. Make a list of twelve categorematic words, three to illustrate each of the four parts of speech that are categorematic.

3. Make a list of nine syncategorematic words, three to illustrate each of the three parts of speech that are syncategorematic.

4. Make five sentences of varying complexity and pick out the terms and copula, listing the terms as first (subject-) and second (predicate-) terms.

5. Analyze the copula from the second term in each of the following sentences; then rewrite the sentences so that the copula and second term are distinct:

> 1. *The moonlight plays fitfully upon yonder rippling stream.*
> 2. *Our own misdeeds speak strongest in our condemnation.*
> 3. *The wind blows from the south-east.*
> 4. *John plays with his school-mates.*
> 5. *John played with his school-mates.*

6. Make a list of three examples of each class of terms noted in the text.

7. Mention five abstract terms that refer to ideas that have tended toward concreteness through hypostasis of abstractions.

8. Arrange five pairs of (true) contradictories under the headings "Positive" and "Negative"; then put down the corresponding terms that were once true contradictories, but are no longer such owing to language changes; as, for example:

CONTRADICTORIES

POSITIVE	NEGATIVE	FALSE NEGATIVE
temperate	non-temperate	intemperate
valuable	non-valuable	invaluable

9. Classify the following terms as in the sample table given below by making a cross in the appropriate column, noting especially that the same term may belong in more than one column:

	NUMERICAL REFERENCE				MODE OF CONCEPTION		MODE OF ASSERTION	
	Singular		General				Contradictories	
	Individual	Collective	Distributive	Collective	Concrete	Abstract	Positive	Negative
desirability						*	*	
racehorse			*		*			
flock				*	*			
untrue					*			*

Words to be arranged

wrath	detachment	Miss Jones	Mars
New York	servant	Maine	indescribable
company	czar	home	President Roosevelt
incompetent	United States	brilliancy	Democratic party
relation	non-support	reckless	Junior A 3
wood	wealth	powder	lightness
independent	dialect	water	moon
nation	psychology	Charles I	insipidity

10. Name ten pairs of relative terms, and indicate which are the correlatives.

11. Name a term closely connected by association with each of the ten terms named in answer to Exercise 10.

CHAPTER III.—AMBIGUITY

13. How Terms Come to Have Different Meanings.—Our ideas are far more numerous than the vocabulary that we command for their expression. Hence we are put to the necessity of using the same terms to symbolize different ideas. This tendency is increased by our habit of using figurative language in poetical and dignified modes of expression. Then again language is full of "faded metaphors"—by which are meant figurative expressions that have, in the course of time, become so trite as to have lost their character as figures of speech. Nearly all our terms for ideas that are not the direct product of sense-perception are of this character. For example, such terms as *impression*, *conception*, etc., reveal in their etymology their character as faded metaphors.

This use of terms to express more than one idea is a great enrichment of our power of expression; for without it many of the ideas we have could not be communicated to others. But unfortunately it greatly increases the liability to confusion of meaning. This is, of course, a logical rather than a grammatical or rhetorical difficulty; and since logic must accept the form of language as given by usage, the only thing it can do is to suggest cautions against ambiguity, and to impose certain prac-

tical rules for securing singleness of meaning as long as a term is employed in one and the same discourse.

14. Definitions.—A univocal term is one *which suggests to the mind only one possible meaning.* There are relatively few such words in the language. Scientific and technical terms furnish the best examples. Indeed, science marks its advance largely by the ability of its followers to determine and establish a *nomenclature, i. e.,* a technical vocabulary that is univocal. Such words as *quinine, arc-light, eau-de-cologne, radium, X-rays,* are univocal.

An ambiguous or equivocal term is one *which suggests to the mind more than one meaning.* Most terms are more or less ambiguous, especially terms referring to objects of thought and imagination rather than of sense-perception. Examples of ambiguous or equivocal terms are *church, spirit, minister, force, faculty.*

15. The Fallacy of Equivocation.—The fallacy of equivocation is *an erroneous meaning which grows out of the use of an ambiguous term.* It may arise either from an unconscious confusion in the user's own mind, or from a malevolent purpose of confusing others. In the latter case it is frequently called a *sophism* or *quibble,* and is a notorious weapon for defending a weak case against an opponent who is too ignorant or too slow-witted to detect the trickery. More will be said about this fallacy in a later chapter on the fallacies. It is interesting to notice that punning and the usual wit and humor that pass current in the newspaper column depend upon that form of ambiguity called "a play upon words."

16. Types of Ambiguity.—A rough distinction may be made between ambiguities of spoken discourse, written discourse, and those incident to both. The words *air* and *heir, heart* and *hart, hair* and *hare,* would be liable to misapprehension only when heard. On the other hand, *tear* (from the eye) and *tear* (a rent in clothing), *lead* (a metal) and *lead* (guidance), would prove confusing only when read. But *rent* (of a house) and *rent* (a tear in clothing), *relation* (colloquialism for relative), *relation* (connection between things), and *relation* (the narrative of a story), might easily mislead the intelligence whether the appeal was to the ear or the eye.

We may almost disregard the first two types as occasions of serious misapprehension of meaning; for they are easily detected and may be corrected by reference to the context or situation. But the last class of words is the cause of serious errors, especially if there is a basis of real likeness between the several things whose meaning the term may suggest.

In addition to the mistakes to which the cultured are liable, there are the legions of misspellings and mispronunciations of the ignorant to throw their scanty store of correct ideas into confusion. But for the fallacies of true ignorance education, and not logic, must supply the medicine.

It should be noted that ambiguity almost never occurs in discourse between words representing different parts of speech. For example, one would not confuse the two words "*bear*" in such a sentence as this: *The fearless hunter levels his gun as the bear bears down upon him.* Under such circumstances the widely differing functions of the words secure to them the necessary singleness of meaning.

It is sufficiently shown, then, that trouble arises nearly altogether when the sound conspires with the spelling to mislead, when the grammatical functions of the words are identical, and when the things to which they refer, be they thought products or material objects, have an underlying ground of real or supposed likeness.

17. The Psychological Basis of Ambiguity.—It is confusion of ideas that lies at the bottom of prac-

tically all serious instances of the ambiguous use of terms. This confusion is accentuated by various accidents in the development of language. A rough grouping of the causes leads to the following classes of ambiguity due to language growth.

First, there is accidental confusion between words that have come into the language from different sources. Here a slight original similarity of sound has caused the words to become moulded into the same form, because the things to which they are applied are a little alike. Our English vocabulary, made up of words from Latin, from Anglo-Saxon, and from the Norman-French, is especially liable to this tendency toward the further assimilation of words that sound slightly alike to begin with. Thus the word *mean* has become ambiguous in the adjective form. In the sentence, *He was a man who always chose a mean line of action,* one cannot be certain whether it is implied that the line of action was low and vicious or that it was cautious and moderate. The word as used in the one sense really comes from the Anglo-Saxon, *gemaene;* in the other, from the Latin, *medianus,* through the Old French, *meien.* The noun *bill* is another instance of the same sort. Meaning an itemized statement, it is derived from the late-Latin, *billa;* meaning a bird's beak, from the Anglo-Saxon, *bile;* meaning a halberd, from the Anglo-Saxon, *bill.*

Second, there is the confusion that arises from the mind's proneness to associate together experiences that occur at the same time or place, or are alike in some respect. The psychologist gives to this tendency the name of *Association of Ideas.* As this process has al-

ready been discussed sufficiently (p. 9), it needs only to be mentioned here as a prolific source of ambiguity. Let things be once experienced together in sense-perception, and the same things tend to remain as features in the memory-image out of which the idea develops. If anything excites one feature of the idea the other factors are also brought up. Thus, if one has heard a new song at the theatre in the company of a dear friend whom he has not seen for a long time, the two impressions become adherent parts of one idea, so that ever after the song will call up the face of the friend. If there is any possible excuse for doing so, it not infrequently happens that the name of one feature of such an idea is also used for the other feature. So in our repeated experience we find church edifices, congregations, and the doctrines which the ceremonial of worship suggests to be associated factors, attention now bringing the one and again the other factor to the foreground of consciousness. The underlying relation is a sufficient warrant for us to transfer the word *church* from the building which shelters the congregation, to the congregation itself, and finally to the body of doctrine which brings them together for worship. In this way we speak of *St. Paul's Church,* say that *church is out,* and allude to the *Church of England* in almost the same breath.

It is cases of this kind that give us our most serious instances of ambiguity, especially when the terms relate to ideas that are well on the road to abstraction.

Third, minor ambiguities occur from transfer of meaning to analogous things where the likeness is felt

rather than clearly perceived. To this cause may be referred all ambiguous epithets that have attained currency through colloquial usage, through efforts at literary effect, and through the authority of poets. Throughout the range of human experience these dimly perceived analogies are to be found, oftentimes uniting for the moment things that are otherwise profoundly unlike. Some of them probably arise from what the psychologist calls fusion of sensation, as when we speak of a sweet smell, using an adjective that properly applies only to the sense of taste. It is probable that in this case the smell of the perfume of a flower does stimulate the nerves of taste to a slight activity. This transfer of epithet is at the basis of all graphic style. Shakespeare uses it with marvellous effect, and all the choir of poets, major and minor, find in it the mainstay of their art. I quote a few such epithets from Shakespeare:

> In the dead vast and middle of the night.
> (*Hamlet,* I, ii, 198.)
>
> Left and abandoned of his velvet friends.
> (*As You Like It,* II, i, 50.)
>
> Your loop'd and window'd raggedness.
> (*King Lear,* III, iv, 31.)
>
> The lean and slipper'd pantaloon.
> (*As You Like It,* II, vii, 158.)

In these lines the words *dead, velvet, looped, windowed,* and *lean* all ordinarily refer to something quite different from their meaning as used by Shakespeare. And yet we all feel in a subtle way that no others could so well have conveyed just the sense which the poet had in mind. This is because we detect an analogy be-

tween the several things here qualified and the objects to which such adjectives usually apply. It is so subtle that Shakespeare's mind was needed to suggest it first; ours can at best merely imitatively follow it.

While these dimly perceived analogies may be of great service in vivifying style, they often lead to no little confusion of thought when pressed into the ordinary vocabulary. Hence the average speaker and writer needs to be on his guard about using them unless firmly fixed by custom. The following examples of such analogies that have acquired general currency will suffice to suggest to the pupil's mind other instances: *dull day, brilliant wit, sharp satire, cutting sarcasm.*

The above discussion as to the causes of the more serious variety of ambiguity leads us to the conclusion that the root of the trouble is confused ideas rather than a misapplication of terms. Such a confusion results from incomplete classification. Everything in the universe can show some few points of likeness to everything else, so that it is possible to classify any two conceivable things together. Permanent classes arise, however, only in answer to some clearly defined need. When such permanent classes are established terms are invented to designate them. These permanent classes are established because we have noticed that the individual things composing them have important, permanent, and numerous characteristics in common. But in addition to these permanent classes or kinds the mind is constantly making loose, temporary, makeshift classes, owing to the fact that some minor point of similarity gets momentary attention, while for the time the profound dissimilarities are ignored.

It is while the mind is on the way to permanent classes, as just described, that it is fond of using the same term for the experiences that are momentarily classed together. But since the unlikenesses are far more numerous and significant than the likenesses in these temporary classes, confusions of all kinds arise as soon as the likeness falls away from the focus of attention and a feeling of the real dissimilarity usurps its place.

18. How to Correct Ambiguity.—To the question, How may this fatal tendency to ambiguity be corrected? there is one obvious answer: make more accurate classes. How this is to be done belongs more appropriately to the chapter on classification than here. In passing it may be said that classes are answers to a need for handling our manifold experiences more economically, and that while our experience is growing and evolving, they must be elastic enough to submit to such modification as enlarging knowledge may, from time to time, determine. Yet with due allowance for changes there may be scientific accuracy within a reasonable degree; for an elastic classification is not an enemy to accuracy so far as we have gone.

A word of suggestion to prospective teachers is here in point. The best practical corrective of ambiguity with young children is referring the term to the objects for which it stands. The wordy and profitless disputes of the mediæval schoolmen would never have been had some Comenius of an earlier generation forced them to point out the realities to which their terms applied. The shibboleth of the educational realists, "*Things before words*," had to be dinned into the ears of many generations of men before the world awakened to the fruitlessness of mere talk as an educating agency. To-day we live in a pedagogical era of pointing out things. It may be hoped that this best and sanest method of correcting ambiguity will produce the results that it should if only the teacher is careful to heed even a better pedagogical formula than that of the realists, viz., "*Things with words.*"

Another means of correcting ambiguity within the teacher's reach is the *context*. Sentences, and even paragraphs, are the units of discourse, and language study must act with a view to this fact. Teach the child to see the true meaning of the term by getting it in its setting. It is rare that careful attention on the part of the reader or listener to what has been previously written or said will not accomplish much toward illuminating an obscure word or phrase. The mechanical glibness with which

most reading in the schools is accomplished, aiming too much at perfection of form, too little at a full understanding of the meaning, is a prolific source of lax habits of thought that multiply the chances of ambiguity. One hears long argumentative harangues on religious, moral, political, and social topics that turn entirely upon a perverse misapprehension of meaning. After the debaters are worn out they usually close by admitting that both meant the same, and that the debate was a useless expenditure of energy; the only trouble being that neither would pause long enough to bring the other's terms in comparison with the context.

To sum this up: let the teacher take every possible advantage of the opportunities to refer terms to the realities that they symbolize; let him utilize pictures if the realities cannot be present in the flesh; let him question and develop paraphrases if neither thing nor picture is available; let him teach reading for understanding's sake rather than for mechanical perfection; and let him, above all, summon the oral and written context to illumine words and phrases that, when standing alone, prove misleading.

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REVIEW QUESTIONS

1. *Why do we need to use the same term for more than one meaning?*
2. *Name the advantage and disadvantage of this.*
3. *Where do we find the best cases of univocal terms?*
4. *What kind of terms is most likely to be ambiguous?*
5. *In what two ways may the fallacy of equivocation be used? What do you understand by a sophism?*
6. *Upon what do punning and the minor forms of wit depend?*

7. *What three conditions conspire to cause the most serious cases of ambiguity.*

8. *Name the three causes of the most serious ambiguities.*

9. *What is the literary value of "transfer of epithet"?*

10. *How does incomplete classification influence ambiguity?*

11. *What means should the teacher employ to avoid and correct ambiguity?*

EXERCISES ON CHAPTER III

1. Find five examples of "faded metaphors" in words of Latin derivation.

2. Make a list of ten univocal terms.

3. Make a list of ten equivocal terms, and show how they are ambiguous.

4. Write five sentences illustrating ambiguous terms.

5. Cite five instances of words that are ambiguous only when heard.

6. Cite five instances of words that are ambiguous only when read.

7. Mention five words that are ambiguous to both ear and eye.

8. In the following list of words show, 1st, wherein lies the ambiguity; 2d, what is the cause (consulting the dictionary for the etymology when necessary):

rate	*bill*	*house*	*fish*	*star*	*law*
rod	*hang*	*play*	*injunction*	*sun*	*lode*
volume	*hare*	*lecture*	*bed*	*school*	*peer*
boss	*branch*	*peal*	*intension*	*tale*	*foot*

9. Make five sentences in which words selected from the above list are used so as to be ambiguous.

10. Cite ten passages from important English poets, such as Shakespeare, Tennyson, and Browning, where transferred epithet occurs, and be prepared to explain the analogy.

CHAPTER IV.—DENOTATION AND CONNOTATION.

19. The Double Function of General Terms.—We have seen how a general term names a class of things that are grouped together, because they all have common qualities or attributes. Now two facts are apparent about such a class: first, it is composed of at least two (usually several or many) individuals; second, no matter what the other varying attributes, there must be at least one (usually more) attribute or characteristic that these individuals have in common. The general name that symbolizes the class must, therefore, do double duty: it must make clear to our minds the fact that individuals compose the class; it must also call to our notice the fact that a common attribute or common attributes bind these individuals together into a class. For example, the general term *dog* points out *Fido, Rover, Sport,* and the millions of unnamed yellow, black, spotted, and indescribable curs that common consent looks upon as coming within this particular class of animals; the same term also brings more or less prominently before the attention those essential common attributes whose presence in certain animals enables us to recognize them as members of this class.

The first of these functions is the *pointing-out function;* the second is the *function of calling up the essential marks of recognition.*

20. DENOTATION.—Denotation means the *sum-total of the individuals of the class to which the general term applies.* Several synonyms for this word are in common use. The most usual of these is the word *extension. Breadth* and *scope* are also occasionally used. *Denotation* is preferable because it has the verb-form, *denote,* the adjective-form, *denotative,* the adverb-form, *denotatively,* and the phrase-form, *in denotation.* It has also the advantage of being more technical and hence less ambiguous than its rivals.

21. CONNOTATION.—Connotation may be defined as *that function of a general term which calls to mind the essential attributes by the possession of which an individual can be referred to its class.* The most common synonym for this term is *intension.* The word *depth* is also frequently used. *Connotation* is preferable both because it is more technical, and because there is the verb-form, *connote,* the adjective-form, *connotative,* the adverb-form *connotatively,* and the phrase-form, *in connotation.*

General terms differ very greatly as to the number of individuals they serve to denote, and also as to the attributes they serve to connote. Terms applying to a class composed of only a few known individuals, such as the word *planet,* can easily have their denotation determined by naming the several individuals; for example, the list, *Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune,* gives us the denotation of the term *planet,* as ordinarily used. But where the class is large or where the individuals composing it change, it is impossible to determine denotation by individuals. Under such circumstances the best that can be done is to name all the sub-classes into which the higher class is divided; thus in the case of the term *dog,* no one would attempt to make a census of all existing individuals of the class; it would suffice for practical purposes to name the sub-classes, *poodle, collie, terrier, hound, Great Dane, bull,* etc.

It is often even more difficult to determine the connotation of a general term. Precise connotation is the aim of all scientific classification; but not infrequently the investigator meets with what may be called *marginal cases, i. e.*, individuals possessing attributes which seem to place them on the border line between two classes rather than definitely in either. For example, a small, fish-like animal called the *amphioxus* (lancelet) exhibits many of the attributes that characterize the invertebrates; yet in other important points it shows the structure of the lowest fishes, which are, of course, vertebrates. The naturalist is here confronted with an interesting problem in classification and nomenclature. Shall he ignore its invertebrate characteristics, or shall he ignore its vertebrate characteristics? Apart from this difficulty in connotation there is the further fact that we often *feel* the marks of recognition by which we name things rather than *know* them. Thus the child recognizes various individuals of the canine family as dogs; but it is scarcely probable that he could name the essential attributes of which the connotation of the term is made up. In spite of these difficulties, or rather because of them, it is an excellent practice to train ourselves to view general terms in both their denotative and connotative functions, remembering that both functions are always more or less present, and should be definitely brought out if we are to know the real *meaning*. In the *connotation* we have the *material of the definition*, and in the *denotation* the *illustration* or *example which gives the concrete reference to the definition.*

22. VARIATION OF DENOTATION AND CONNOTATION. —There is a certain rough inverse relation between the denotation and the connotation of terms that may be expressed by saying that *the wider the range of application of a term, the fewer are the attributes that the individuals to which it refers will share in common.* For example, it must be obvious on the slightest reflection that a term which is a general name for all Europeans, though referring to more persons, will imply less characteristics in common than a term which names only one nation, say English. In passing from a term referring to a more numerous class to one that can apply only to

a less numerous sub-group of that class, we are obviously cutting down the number of individuals to which it is applicable; but, just because they are fewer, they have more characteristics in common. The easiest way to show this is by taking a term and applying to it a number of qualifying adjectives, which, though adding attributes, cut down the things to which the term is applicable. Take, for example, the term *man.* Unqualified, it applies to all the inhabitants of our globe, but they share only in the attributes of *upright stature, rationality,* etc. Now prefix the adjective *white;* we have added to the connotation, but have immensely limited the denotation. Next prefix the adjective *European* to the term *white man,* and the range of application is still further narrowed, while the connotation is increased. *French* before *European white man* carries the process a step further, and *Parisian* before *French European white man* makes a final step toward limiting the denotation and increasing the connotation. The following table assists the eye to realize this relation:

	DENOTATION	CONNOTATION
man.	*Total population of the earth.*	*Upright stature.* *Rationality.*
white man. (*Caucasian*)	*White population of the earth.*	*Upright stature.* *Rationality.* *White skin.*
European white man. (*European*)	*White population of Europe.*	*Upright stature.* *Rationality.* *White skin.* *Inhabiting Europe.*

French European white man.	*(Frenchman)*	*White population of France.*	*Upright stature.* *Rationality.* *White skin.* *Inhabiting Europe.* *Inhabiting France.*
Parisian French European white man.	*(Parisian)*	*White population of Paris.*	*Upright stature.* *Rationality.* *White skin.* *Inhabiting Europe.* *Inhabiting France.* *Inhabiting Paris.*

A useful exercise to help the student in understanding the relation of the connotation and denotation of terms is to select a series of terms which name higher and lower classes for arrangement in orderly sequence so as to show either increasing or decreasing connotation or denotation; then the student is to show how such an arrangement affects either the denotation or connotation. It must be noted that the terms selected *must stand in the relation of higher and lower classes.*

A series of terms varying in denotation and connotation may show the variation either by a complete change of word or by prefixing modifiers to a constant term. Examples of the former are: *man, European, German, Prussian, Berliner;* of the latter: *man, white man, American white man, cultured American white man.*

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REVIEW QUESTIONS

1. *What two things are necessary in every class?*
2. *What two corresponding functions has the general term?*
3. *State the definition of denotation.*
4. *What are synonyms for this term?*
5. *Why is it preferable to its synonyms?*
6. *Define connotation.*
7. *Name synonyms of connotation.*
8. *How would you express the denotation when the class is small? when large?*
9. *Why does science aim at precise connotation?*
10. *What do you understand by "marginal cases"? Name an instance from some experience of your own.*
11. *What is the relation between denotation and connotation?*
12. *What must be the relation between terms used in an exercise to show changing denotation or connotation?*
13. *In what ways may the form of terms be made to show changing connotation?*

EXERCISES ON CHAPTER IV

1. In one column indicate the denotation (by species when the class is large) and in the other the connotation (consult the dictionary, encyclopædia, or science text-book if necessary) of the following terms: *continent, mountain system, moon, ocean, swan.*
2. Fix according to the above directions the denotation and connotation of three general terms of your own choosing.
3. Arrange the following terms in order of increasing denotation, being prepared to show how the connotation is affected in so arranging them: *president, Benjamin Harrison, Republican president, native-born citizen of the United States, Republican president from Indiana, white man, man, American, American statesman.*
4. Arrange the above terms in order of decreasing denotation, increasing connotation, and decreasing connotation, being prepared to show how each arrangement influences connotation or denotation.

5. Make your own single series of five terms and arrange in four lists, headed: *Increasing Denotation, Decreasing Connotation, Decreasing Denotation, Increasing Connotation.* How are the lists related?

PART II.—CLASSES

CHAPTER V.—CLASSIFICATION AND DIVISION

23. Classification as a Process.—Thus far our study of logic has shown us that general terms are the symbols of classes. We must now turn our attention to the process of classification. As a process, classification is psychological; for it is a special manner of the mind's operation. But the function of the process is distinctly logical; for it aims to get our experiences grouped so that we can better determine their meaning. Our minds always tend to *unify* their experiences. Indeed, it is doubtful if experience would have any meaning at all were it not for this unity which the classifying process brings about. Experience itself, if we mean by it more than being momentarily conscious while something happens, is a unity—the result of a species of classification; for each element of it must be brought into a group-relation with each other element, and running through the elements must be detected the common characteristic that it is *my* experience.

As the result of the process of classification, we get classes or kinds, *i. e.*, groups of units or elements in all of which we detect a bond of likeness no matter what their diversities in other points. Logic must assume that we can classify because objective likenesses exist

among the facts of our personal experience, whether they be material realities or the no less real thoughts and fancies to which we ascribe merely mental existence. We find all degrees of likeness of characteristic. On the one hand is the identity in form, size, color, and material found in ten marbles—where the only noticeable difference is their occupancy of different parts of space. On the other is the common attribute of materiality, binding together in a class two things otherwise as remote as a man and a stone.

But however things may be in their own nature, there is a certain feature of will and purpose about our classifying of them. It is *we* that group them as we do—though never completely ignoring *their* objective affinities. And we group them always in view of some end beyond classification itself. This ulterior end is usually *explanation*. We can explain only after we have determined the points of identity in a group of objects; for it is not till identity has been detected that we have any ground of connection between the things requiring explanation. After we have noted in what points the things of a group are alike, we next describe in terms of the common attributes. If the common properties are numerous and important, our description finished, we set out to find the reason for the common qualities. This reason may be regarded either from the stand-point of a condition out of which the common properties arose or of a purpose for which they exist. In the former case we have a *causal explanation*, in the latter a *teleological explanation*—the one the explanation that satisfies exact science, the other that demanded by philosophy.

Thus it appears that classification is a preliminary to determining the cause which will produce the kind of things under investigation, and, where possible, the purpose for which they exist.

24. Definition of Classification.—Classification is *the process of grouping things together on the basis of common attributes.* It is a psychological process which meets a logical need. We symbolize the classes which we get by *class-names* or *general terms.* The mental counterpart of such classes is the *class-notion, concept, general notion,* or *general idea* (all these terms are used to refer to it).

25. Artificial and Natural Classification.—It is important to distinguish between *artificial* and *natural* classification. The former selects as the bond of union some characteristic that is of little real importance to the nature of the group obtained; the latter selects a characteristic that is essential to the nature of the group obtained, and usually one that is correlated with other characteristics. The cataloguing of books according to the initial of the authors' names exemplifies artificial classification; whereas the classification of certain animals as vertebrates illustrates a natural classification. In the one case little or nothing goes with the classification; in the other, much does. The subject-matter of the books, the size, the literary style—all these things are left in doubt; but when we know that a certain group of animals have backbones, we are prepared to find numerous other important attributes accompanying and interdependent with this structure—such as internal skeleton, type of nervous system, etc.

Artificial classifications are of service in furthering some interest we have for the time being, such as making the things so grouped easier to deal with. Natural classifications answer to the real nature of the things so grouped, and give us an insight into the permanent nature and relationships of objects of scientific interest. Hence artificial classifications are non-scientific; natural classifications, scientific.

26. Influence of the Theory of Evolution upon Scientific Classification.—Scientific classification was once a very ambitious procedure, boasting that ultimately perfect and changeless classes might be obtained. The doctrine of evolution, particularly as applied to biology, has greatly modified these claims. The old-fashioned theory which evolution has displaced held to the belief that *species* (*classes*) *are immutable, i. e.*, that natural kinds do not shade off into each other. Evolution has accumulated a vast amount of evidence that just the opposite is true, and where evidence is lacking hypothesis is brought in to extend the principle of "missing links" to all needed cases. Thus it has come about that it is possible to supply, in thought at least, a bridge of intermediate links between species as now known, and also between higher forms that now exist and their lower prehistoric progenitors. Consistently carried out, this would make classes merely convenient groups rather than fundamentally different *kinds*.

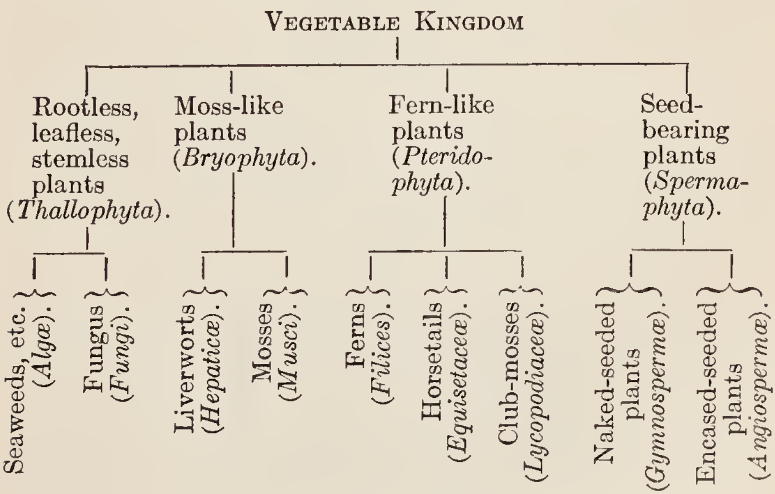
From this point of view one can see that all conceivable classes and systems of classifications are destined to change with the development and enlargement of

knowledge. Indeed, classification itself is subject to evolution. This is especially seen in scientific classifications, which are continually being modified with the growth of knowledge. It appears from this that no groups are permanent. Classes are no more than made until they must be either modified or abandoned. This is the price that we have to pay for mental growth and development—the work is no sooner done than it must be repeated. Let no one, then, be deluded into believing that truth is static and that our data may be classified for all time to come. Our notions—and classes are merely a special kind of general notion or concept—change with our experience and with that of the race. We are in an eternal quest for changeless truths, but are destined never to find them.

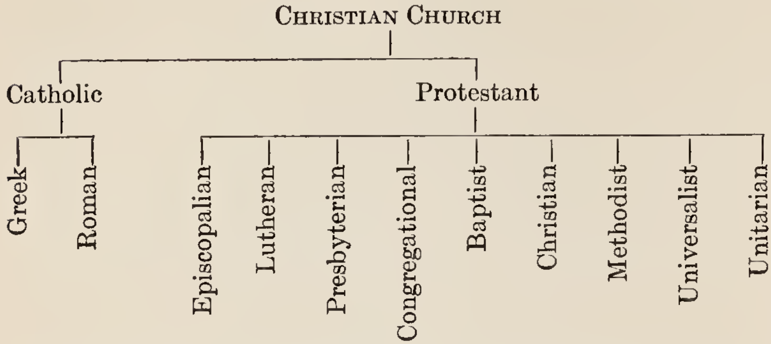
27. Organization of Classes in a Hierarchy or System.—Classes serve a double function, first, to include a number of individuals in which a bond of likeness has been made out, and second, to exhibit the relationship existing between such groups of individuals. The former function has already received sufficient attention; the latter needs to be enlarged upon. Our minds work upon the presumption that all experience is a unity, that things, facts, and mental states are all immediately or remotely connected and interdependent. This assumption is expressed in a thousand ways. It lurks in the background of our thought when we say that our *experience* tells us that such a thing is true or untrue; when we assert that natural happenings are all parts of *nature*. Experience, nature, etc., all of them are abstract terms that we use to include immense groups

of fact. Such terms imply that our facts all belong together or form parts of one organization. Such an organization is called a *system.* Classes must not only include the individuals which fall within them; they must also show how they are related to other classes, either as subordinate or co-ordinate members of a hierarchy or scheme of classes. Only when such organization has been made do we have true relationships established. Systems of classification in such natural sciences as botany and zoölogy illustrate hierarchies of the kind referred to here. With the classes so arranged in subordination and co-ordination, we are able to see at a glance what relation one instance or fact has to all others. The truer and better the organization, the more complete is our knowledge of the facts comprehended within the classes.

The following classification of plants, which is favored by many recent botanists, exhibits such a system:



Another example of a system of classes is the following:



28. DIVISION.—Classification is a unifying process. Opposed to it is *division,* which is essentially a discriminating or separating process. The one is *synthetic,* the other *analytic.* Yet, when all is said and done, the difference between the two really implies a close relation. For when our classes are arranged in a perfectly organized system which reveals their true co-ordination and subordination we already have a scheme of division. Thus in the last example given on the previous page the system of classes as there organized is identical with the division of the higher class, "*Christian Church,*" into the subordinate classes which go to constitute it. This may be summed up as follows: *Going from the smaller classes toward the larger involves classification; from the larger toward the smaller, division.*

29. LOGICAL DIVISION.—The kind of division just mentioned is called *Logical Division.* It may be defined as *the process of differentiating minor groups* (*ultimately it gives individuals*) *within a class by detecting variations in some attribute.* The sub-classes are obtained by discovering characteristics that some individuals

have and others do not have. The principle used in dividing is called the *ground of division* (*fundamentum divisionis*).

30. Rules Governing Logical Division.—To be correct, Logical Division must conform to the following rules:

I. Rule of Completeness: *The sub-groups* (*species*) *must suffice to provide places for all the individuals of the class divided* (*genus*); *i. e., the sum-total of the several denotations of the species must just equal the denotation of the genus divided.*

The purpose of this rule is at once apparent when we consider the function of division, viz., to differentiate all the members of a class according to varying attributes. No class is completely divided unless all the individuals known to belong to it are provided for, and if all are provided for there will be an exact balance between the sum of the individuals regrouped in the species and the number of individuals in the genus which was to be divided.

II. Rule of Exclusion: *The sub-groups* (*species*) *into which we divide the higher class* (*genus*) *must exclude one another; i. e., no individual may belong in more than one species.*

This rule aims to prevent the *overlapping of species*. If we divide horses into race-horses, carriage-horses, and broken horses we are guilty of the fallacy of overlapping species; for the last species, broken horses, would include at least some individuals belonging to the others.

It is scarcely possible to apply this rule rigidly in most instances of the division of natural phenomena. As a rule, the distinctions between individuals shade off so gradually that the naturalist is perplexed as to what disposition he is to make of a given case. The *lancelet*, already cited as an illustration on page 35, is an instance in point here also. It may be questioned whether in dividing we should not put this form into two species, because of the peculiar entanglement of attributes.

III. Rule of Consistency of Ground: *The division must be completed by applying only one ground of division (fundamentum divisionis).*

This rule is so closely connected with the foregoing as to seem almost identical with it. To change the ground of division would give us a result illustrated in the following case, where we change while dividing from the *structure of the flower* (which determines the *kind*) to its *color:*

Cultivated flowers

Roses | Geraniums | Red flowers | Purple flowers

Changing the ground while dividing results in the fallacy of *cross division,* and produces *overlapping species.*

31. Changes in the Classification and Division of the Same Data.—The same data may be classified in many different ways, depending upon the point of view and purpose of the classifier. Likewise the same class may be divided in various ways, depending upon the application of different grounds of division. Suppose, for instance, we wish to divide the class *triangle* into the minor groups of figures which constitute it. We may take as our *fundamentum divisionis* the relative size of the enclosing lines. We should then get the following result:

Triangle

Equilateral | Isosceles | Scalene

Or we might use as the *fundamentum divisionis* the included angle, and we should have another division:

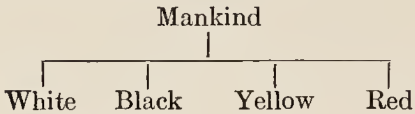
Triangle

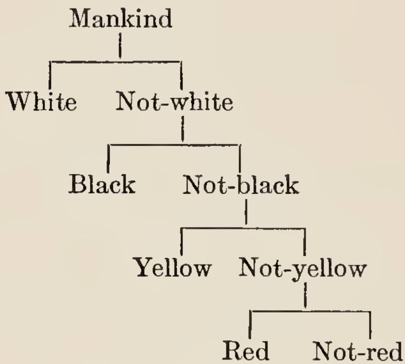
Right-angled | Equiangular | Obtuse-angled

32. Logical Division as Related to Denotation. —In logical division we differentiate a class into its constituent sub-classes. This process it is possible to keep up until sub-classes are found which, upon differentiation, give individuals. The mind goes through much the same procedure in seeking to determine the denotation of a term; although in this case we are usually content to pass at once to the individuals, ignoring intermediate classes. We have oftentimes to give the denotation of a term by naming the sub-classes included under the class of which it is the symbol. Under such circumstances logical division is identical with finding a term's denotation. It is necessary to notice, however, that we *divide* a *class* but give the *denotation* of a *term*, though in loose speech the two points are often confused.

33. Dichotomy.—The aim of division is to provide a place for every individual in the class divided; yet it may sometimes happen through oversight, or owing to the growth of the class, that no appropriate sub-class has been provided for part of the class. To prevent such a contingency is the aim of a tedious process of two-fold division known as *dichotomy*. This may be defined as *the process of dividing a genus into two species on the ground of contradictory attributes*. The need of dichotomy arises when one is not sure of the limits of the class. If, for instance, we divide the human race according to the variation in color into white, black, yellow, and red, we run the risk that there may be discovered in some region at present inaccessible a race with skin of different color from any so far known; and for such our logical division provides no place. Dichotomy or *ex-*

haustive division, as some call it, forestalls the difficulty by making two sub-classes, one having an attribute and the other its contradictory (see p. 18). The following divisions, the one logical and the other dichotomy, show how much longer the latter is.





Two other kinds of analysis should be distinguished from both of the processes just considered, viz., *partition* and *metaphysical division.* The former consists in *analyzing an organic whole into its interdependent parts or organs,* as when we think of a tree as consisting of roots, stem, trunk, leaves, etc. The latter consists in *abstracting an attribute from the concrete thing in connection with which we experience it,* as when we think of the color or fragrance of a rose apart from the rose itself. Metaphysical division yields abstract ideas.

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REVIEW QUESTIONS

1. *What is the logical function of classification?*
2. *How is classification connected with explanation?*
3. *Define classification; class-name; class-notion.*
4. *How would you distinguish artificial from natural classifications?*
5. *How has scientific classification been modified by the theory of evolution?*
6. *What is the value of organizing classes into a system?*
7. *How is division related to classification?*
8. *Define logical division; dichotomy.*
9. *How is logical division related to denotation?*
10. *Explain partition; metaphysical division.*

EXERCISES ON CHAPTER V

1. Group together in a class ten slightly different common objects; then name the common properties used in making the class; next name the more striking differences exhibited by the objects.
2. Group together ten children from an imaginary (or real) school-room; then state the characteristic or characteristics common to all of them; finally, indicate points in which they differ.
3. Cite an instance of your own of an artificial and a natural classification.
4. Select a few groups or classes of related things, and try to organize the classes into a system that will clearly exhibit the true relations.
5. Divide the following classes in two ways: *flying animals; citizens of Brooklyn; school buildings; public schools; teachers; pupils.*
6. Divide the classes given in exercise 5 by dichotomy.
7. Apply partition to the following: *a violet; a desk.*
8. Apply metaphysical division to each of the instances given in exercise 7.

CHAPTER VI.—NAMING, DEFINITION, AND THE PREDICABLES

34. NAMING.—Our classes having been determined, we have next to fit them out with names. This leads to the process of naming, to which the more dignified term *denomination* is usually given. From the stand-point of the student who has accepted a vocabulary as a heritage from the past, the process looks easy enough. All that seems necessary is to call a class by its accepted appellation. But we must remember that what we so glibly use was wrung out of the slow inventive imagination of our more primitive ancestors. They had to *make* the terms *horse, dog, dear, earth,* etc. It belongs rather to linguistics than to logic to trace the romantic history of the growth of language. As logicians, we are concerned with the use rather than the making of terms. We shall have said sufficient on the inventive aspect of the subject if we call attention to the more usual ways in which new terms are got for new experiences. German, a language that compounds with admirable facility, has the advantage of always having at hand a means of meeting every emergency. Let a new discovery be made, and the savant who wishes to describe it has only to combine in a compound two native words already in current use. English is by no means so

facile a tongue. Combinations do not form in it so readily, and are very slow in gaining acceptance. Hence it is that our scientific terms are mainly from the Greek and Latin. The Latin has the comparative disadvantage of having made such a wholesale contribution to our general vocabulary that very few words can be found that have not already established themselves in the language. Greek is better in this respect, but is handicapped by the fact that it is not especially adapted to the English pronunciation, and that it is becoming daily more unfamiliar even to scholars through the neglect which it has suffered in recent secondary and collegiate courses of study.

35. Function of Naming.—Names—especially general ones—are useful in many ways. From a logical stand-point their main function is to convey meaning. By this is meant their function of calling up in another mind an idea corresponding to that the speaker desires to convey. In conveying meaning general terms call into play their two great functions, denotation and connotation. The one gives the range of application, and the other the qualities that must lie in the concept aroused in the hearer's mind if an adequate idea is aroused. In this way the term serves to focus the thought of both speaker and hearer. The speaker renders his thinking clearer and more to the point in the successful prosecution of his search for the right word; the hearer has an adequate idea aroused when the word calls up through association an image of the thing or concept which the word signifies. Another function of terms is as an instrument of record. Were

it not for written language history would be mere hazy guesswork.

In order that the name may suffice for these functions, it should be as brief as possible, and special to the thing or class named. Brevity in words is for reasons of economy of effort. Much time would be saved if our words were shorter. That the name should be special is a necessity if we are to avoid ambiguity, which, on last analysis, is due to the same verbal sign standing for two or more things, and thus arousing two or more possible ideas. Some logicians add another requirement for a serviceable name, that it should imply a description. Perhaps upon analysis this will be found to be rather an invariable characteristic of names than a character found in some and absent in others. For if it be true of a term that it implies the qualities of the class of which it is the sign, as seems to be clearly demonstrated in its connotative function, then it must of necessity imply a description; that, namely, which would render clear the essential qualities found in the class of things which it names. There is, however, a great difference in names in regard to their power of making such implication clearly and quickly apparent. Some languages excel in vocabularies that are technical and yet descriptive. German is such a language. To those who know Greek the descriptions implied in the technical vocabulary borrowed from that language are immediately evident. For example, such names for modern inventions as *telephone, telegraph, microscope, telescope,* etc., readily convey to the mind of one whose Greek has not altogether forsaken him ample ideas of

the use to which the instrument is put. Wherever we can make a new name that renders the implied connotation quickly apparent we should do so.

36. Definition.—Logical definition is the process of *determining the denotation of a class- or general-name, by establishing the essential attributes common to the individuals constituting the class.* Or, more briefly, it is *fixing the boundaries of a class by summing up the essential attributes, i. e., giving the connotation.* Properly speaking, classification and division determine what our classes shall be, while definition establishes the defining attributes. But to state all the attributes in language would, in most cases, be to prolong the definition into a description. Hence we usually content ourselves with a short-hand procedure, viz., naming the *proximate genus* (next higher class), which refers the class being defined to the next higher class, and so implies the generic attributes; and the *difference,* which serves to distinguish the group as a species from its co-ordinate class. It should be noticed that in logical definition we emphasize the connotation of the term defined. Hence the relation between connotation and definition is so close that one part of the definition implies the connotation of the genus of the defined class, while the other part states the distinction between the defined class and its co-ordinate species. The first part of the definition has been said to imply the *conferentia* (the attributes binding the species together into a genus). The second part plainly states the *differentia.*

Oftentimes it aids to clear one's idea of the class defined if he calls up the denotation of the term by naming typical examples

of the class. Such examples should be reprcsentative of the range of individuals belonging to the class. Thus, if one were defining the term *felidœ* (*cat-family*) and sought to aid the thought by citing examples, he should name such as the *domestic cat, lynx, lion, tiger, jaguar,* etc. All dictionaries define both by connotation (proximate genus and difference) and by denotation (pointing out typical examples), as is illustrated by the following quoted definition of the genus, *Insecta*: "*A class of arthropods* (proximate genus) . . . *often restricted to the hexapods* (in this word the difference is implied rather than clearly stated). *Hexapods embrace* (1) *Hymenoptera,* (2) *Diptera,* (3) *Aphaniptera,* (4) *Lepidoptera,* etc." (Examples are here given by naming sub-species.)

37. Suggestions to be Observed in Defining.— The following suggestions will be found useful both in formulating definitions and in testing the value of definitions that are met with in reading and study. They should not be construed as invariable rules from which departure is never permissible, but rather as practical suggestions tending to secure clearness and accuracy.

1. *The definition should imply the essential attributes or qualities of the class defined; hence it should name the next higher class* (*proximate genus*) *and the attribute or group of attributes which distinguish the class defined from the co-ordinate class* (*such attribute or group constitutes the difference*).

2. *The name defined should not be used in defining; nor should any synonym of it be used unless, as occasionally happens, the meaning of the synonym is much better known than the name defined.* The violation of this suggestion leads to a kind of fallacy called *defining in a circle* (*circulus in definiendo*).

3. *The definition should not be so stated as to admit of either more or less individuals than belong to the class; in other words, the denotation allowed by the defining terms should be the same as the denotation of the term defined.*

4. *It is better to state the definition in positive terms whenever possible.* This cannot always be done. In science, for instance, we arc often in a position to say what a thing is not, though to say just what it is would be impossible in the present state of knowledge. Under such circumstances it may be doubted

whether we can really define at all. Yet our ideas are certainly made clearer by the negative statement; and so far forth it seems justifiable to say what a thing is not if we are not in a position to say what it is.

5. *We should not define in ambiguous, obscure, unfamiliar, or figurative language.* Definition is supposed to render a service to our thinking by giving us clearer ideas than we should otherwise have had. Such being its purpose, it is obvious that we defeat our end if the defining language is liable to more than one interpretation, if it is even less clear to the understanding than the term we are defining, and if it is not familiar. Figurative language may occasionally be defended, because while it does not strictly define, it at least suggests an analogy that often illuminates our thinking.

6. *A definition should be as brief as is consistent with clearness and completeness.* Perhaps it would be no exaggeration to say that violations of this suggestion lead to more word battles than errors of any other kind. Treatises are written on controverted points when the whole dispute arose out of a definition that was so long-drawn-out as to lead to the utter rout of the understanding. The very intent and purpose of defining is brevity. Were treatises wanted when we are trying to define we should consult them. But they are not wanted. A defining line between classes is the desideratum. The old adage, *Brevity is the soul of wit,* might very well be paraphrased, *Brevity is the soul of the definition.*

38. PREDICABLES.—The stamp of mediævalism with which logic is so often charged is nowhere more conspicuous than in this word. Trace it back to its Latin origin and it clearly says, That which may be asserted. The scholastic logicians thought that some peculiar sanctity attached to the five predicables; that they were universal forms of predicate which could be pronounced of any conceivable subject. In reality they are mere technical terms, two of them, *genus* and *species,* designating classes, and the others, *difference* (*differentia*), *property* (*proprium*), and *accident* (*accidens*), naming qualities or attributes that have certain special characters or functions to be named in defining them.

A genus is *any class considered as divisible into two or more sub-classes (species), i. e., it is the whole to be divided.*

Corresponding to the noun *genus* is the adjective *generic.* Thus we speak of *generic attributes* or *qualities,* and by these are meant those qualities that are found in all the individuals making up the larger class or genus.

A species is *any sub-class obtained as a result of dividing the genus.*

Corresponding to the noun *species* is the adjective *specific.* And by *specific attributes* are meant those that are found throughout the sub-class but not throughout the genus.

A property is *any attribute found in all the individuals of a class, and regarded as necessary in order to constitute that class. Generic properties are those essential to the larger class (genus); specific properties those essential to the smaller class (species).*

A difference is *any property exclusively found in one of the species of a genus, hence serving to differentiate that species from co-ordinate species.* It is obviously found in one species of a genus, but is absent from all other species of co-ordinate rank under the same genus.

Differentia (difference) must be distinguished from *conferentia.* The one distinguishes species; hence is used in dividing. The other unites species into their comprehending genus; hence is used in classification. Both conferentia and differentia are essential to the species of a genus; conferentia only to the genus.

An accident is *an attribute which may or may not be present in a class without in any wise disturbing the properties of the class.*

The *summum genus* is *the highest genus, i. e., one that may not in turn be a species.*

The *infima species* is *the lowest species, i. e., one that may not in turn be a genus, but that is divisible only into individuals.*

Examples: genus=*man;* species=*white man, black man,* etc.; conferentia=*possessing reason;* difference=*having white skin, black skin,* etc.; property=*rationality;* accident=*habitat.*

39. Naming and Definition from the Pedagogical Stand-point.—We cannot discharge our obligations to this somewhat dry topic without a word as to the pedagogical aspect of naming and definition. First as to naming. If we traverse the history of educational theory we are met with the cry of the earlier modern reformers that too much of the preceding teaching was a mere empty reiteration of sounds that were almost meaningless because the things which they signified were not presented first. "*Things before words*" was the motto from Rabelais to Comenius. Perhaps we could emend the motto by stating it in the form, "*Things with words.*" Perception, remembering, and conception should all move on apace with naming in the child's early education. In a word, language, which consists so largely of names, should march steadily on with all the child's mental activities. Two or three suggestions may here be in point. In the first place, the names which enter into the child's early vocabulary must be simple, familiar, frequently used, and short. To introduce long words taxes his feeble powers of pronunciation. Unless words are frequently used and

familiar by reason of having been heard they will be unattended to and meaningless. And we all know that use makes language.

In the second place, the words of the early vocabulary should be such as name things rather than semi-abstract or abstract ideas. *Beautiful* may mean much to the young child; *beauty* can mean little. In the third place, the word should be used and called forth by the teacher in connection with the thing it names or with the memory-image or idea of that thing; the latter may be done in composition work, story-telling, etc. In the fourth place, progress should be secured in leading the child by easy gradations to the names of less familiar and more abstract ideas. Teachers often mistake mere glib utterance of the familiar for genuine progress. While the child's limitations must be recognized, that system of training which keeps him always upon the treadmill of the easy and habitual is to be condemned. Be sure of development from a simpler and less technical to a more technical vocabulary. In the fifth place, accuracy in naming should always be an object of the teacher's endeavor. No greater mistake can be made than to think that mere verbal expression is desirable. Not mere idle chatter, but expression which *means something*, is the goal of language-study. No better drill in accurate modes of expression can be found than to have the child associate the name with the object and detect the ambiguities which the name may harbor. In this way habits of precision, both in thought and expression, will be formed—habits that are of inestimable service in after life. For without precision of thought there is no pre-

cision of action; hence no stability of moral character. Two-thirds of the world's liars and thieves are as they are because they have learned neither to think nor to talk straight.

As for definition, it is admittedly one of the most important phases of education. Young children cannot be expected to define with any high degree of accuracy. Yet they may in a certain sense render their experiences clearer by grouping them according to their common properties. Such groupings, involving rough classifying, are the indispensable conditions of defining. The first crude definitions made, the rest of the process is a matter of refining upon the earlier attempts, and of finding language that is precise enough to render the idea in the definer's mind.

The upshot of the whole matter is that things and ideas must be presented before the definitions can be obtained, and that all fruitful defining must be the product of the pupil's self-activity under the guidance and suggestion of the teacher. Let the things, ideas, or what not, requiring definition, be first presented; let them next be classified so as to make clear the essential agreements and differences; let a statement of these essential agreements and differences be made in the pupil's own words; and let the refinements and abridgments proceed under the teacher's skilful guidance and suggestion. Almost before it is expected a good working definition will be forthcoming, and it will have been got through an enlistment of the child's own initiative and interest. Thus the pupil will know the joy of discovery, in coming to a clear knowledge of what a phase of his

own experience really *is* and *means,* not for another, and in the language of another, but for *himself* and in the *language of himself.*

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REVIEW QUESTIONS

1. *Why is naming harder than it seems on first consideration?*
2. *How do languages differ as to their serviceability for naming new things?*
3. *To what languages do we go in the main for our technical terms?*
4. *What is meant by the statement that names convey meaning?*
5. *What other functions have they?*
6. *Name the qualities of a good name.*
7. *Define definition.*
8. *What is the logical method of abbreviating a definition?*
9. *How is definition related to connotation?*
10. *How do dictionaries usually define?*
11. *Be prepared to discuss each of the suggestions to be observed in defining. Do you think of any others?*
12. *How should the predicables be understood?*
13. *How should naming be treated in early education?*
14. *What suggestions can you make as to the kind of names that should enter into the child's early vocabulary?*

15. *How should a vocabulary be developed?*

16. *What is the proper order of introducing definition into early school work?*

EXERCISES ON CHAPTER VI

1. Divide the following genera into species, noting the difference, a property, and an accident of each species: *building, church, population of your home city, teacher, pupil, minister.*

2. Take any five general terms you choose and define them logically (consult the dictionary if you cannot otherwise determine the proximate genus and the difference). Point out typical examples of each class defined.

3. Select from the dictionary ten definitions; point out the proximate genus, the difference, the examples. Criticise them from the logical point of view.

4. Examine the following definitions critically, determine wherein they conform to the suggestions for defining, wherein they violate them, and then correct the definitions:

> (*a*) *An instinct is a propensity prior to experience and independent of instruction.* (*Paley.*)
>
> (*b*) *Thought is the act or product of the Understanding or Reason as distinct from the various processes of simple Apprehension or Cognition.* (*Hyslop.*)
>
> (*c*) *Logic is "the Art of Thinking."* (*Watts.*)
>
> (*d*) *Truth is "that part of human thought which is proved true."* (*Welton.*)
>
> (*e*) *Thinking may be defined in one of its aspects at least as the process of interpreting the special by the general, or the new experience by the old.* (*Hibben.*)
>
> (*f*) *Evolution is an integration of matter and concomitant dissipation of motion, during which the matter passes from an indefinite, incoherent homogeneity to a definite, coherent heterogeneity, and during which the retained motion undergoes a parallel transformation.* (*Spencer.*)
>
> (*g*) *Nature-study is that branch of elementary science which comprehends the study of natural phenomena.*
>
> (*h*) *Psychology is the Science of Mental Life.* (*James.*)

(*i*) *Psychology is the science of the processes whereby an individual becomes aware of a world of objects and adjusts his actions accordingly.* (*Stout.*)

(*j*) *Psychology is the science of the phenomena of consciousness.* (*Baldwin.*)

(*k*) *Psychology is the science of consciousness.* (*Angell.*)

(*l*) *Education is "a gradual adjustment to the spiritual possessions of the race."* (*Butler.*)

(*m*) *Education is the eternal process of superior adjustment of the physically and mentally developed, free, conscious, human being to God, as manifested in the intellectual, emotional, and volitional environment of man.* (*Horne.*)

(*n*) *Education is conscious or voluntary evolution.* (*Davidson.*)

(*o*) *Education consists in leading man, as a thinking, intelligent being, growing into self-consciousness, to a pure and unsullied, conscious and free representation of the inner law of divine Unity.* (*Froebel.*)

(*p*) *Self-activity is activity determined by one's own motives, arising out of one's own interests, sustained by one's own power.* (*Monroe.*)

(*q*) *Self-activity is that phase of activity which arises from the native propensities and develops the conscious needs of the self.*

PART III.—JUDGMENTS AND PROPOSITIONS

CHAPTER VII.—JUDGMENT AS THE LOGICAL UNIT

40. Psychological Evolution of Judgment.—We have now to recur to a subject that has already occupied our attention in the introductory chapter, viz., the judging process. It is necessary to take it up in more detail, for an understanding of it conditions the whole subject of logic. Indeed, so true is this last statement that one would not go amiss to define logic as *the science of valid judging.*

The germ of judgment appears in the very beginning of mental life. The child's first sense of a world around him implies numerous acts of judgment. The awakening out of the lethargy of the prenatal vegetative existence to the vague consciousness of environment constitutes a series of crude judging acts. The reference of the source of infantile pains and pleasures to an outer order of changes contains implicit judgment. Again, the process of concept-forming can scarcely be explained in any other way than by supposing that each repetition of an experience masses with the memory-images of like past experiences, through the agency of naïve judgments of similarity. It is thus that a concept gradually acquires its generality with continued repetitions of like

experiences. But each increment of generality appears to depend upon the recognition by a tacit or expressed judgment of the essential likeness of the new instance of the experience to the old conceptual background. Thus concepts grow by innumerable acts of judgment which assert practical, even if not absolute, identity of nature.

At a slightly later stage in the evolution of the child's mind—a stage that presupposes a small equipment of immature concepts—judging becomes a relating activity that thinks connections between concepts. It is when this stage is reached that logic shows a legitimate interest in the process. For such relating of concepts weaves the threads of partial and incomplete experiences into the pattern of *meaning*, which outlines our real world of truth and action. The introspective study of the judging function just made will become clearer by resort to a typical instance.

Suppose that by increments of experience grafting new instances upon a developing memory-image the child has established a working concept of the class *horse*. Certain prominent characteristics of the horse's behavior, such as *gentleness* and *strength*, have been analyzed out of renewed experiences. The concepts *gentleness* and *strength* have been enriched by experiences of other things than horses. We may assume that these three concepts, *horse*, *strength*, and *gentleness*, would by this time be independent items in the inventory of the child's mental equipment. As such, they could be united in his thinking either affirmatively or negatively. The statement of these judgments would be in the form of propositions to the effect that *horses are strong*, *horses are gentle*, or *horses are strong and gentle*. Now such judgments, rendered objective and real either in speech or action, are proper incitements either to challenge or acceptance on the part of others. They claim extramental validity. They *mean* to assert *universal reality* or *truth*. Stamped with such pretensions logic has a claim upon them, criticises them detects their flaws,

exploits their truth, rejects or accepts them. Assume now that the child has experience only of gentle horses; his confiding nature, his human proneness to generalize too quickly, leads him to the belief that *all horses are gentle.* Acting upon this belief he treats future horses encountered *as though they were gentle* until an enlarged experience brings him into contact with an instance to the contrary, when he modifies the judgment in which the concepts *horse* and *gentle* are united so as to accord more nearly with fact. This modification changes the *meaning* of the concepts *horse* and *gentle,* and produces a corresponding change in the forms of judgment which unite the concepts. Instead of the single affirmative judgment, *(All) horses are gentle,* maturing experience now requires two judgments to express the meaning, one affirmative and the other negative. Truth is now represented in the two forms, *Some horses are gentle, Some horses aren't gentle.* This truer expression of meaning through the judgment has an important consequence—the one vital consequence to the child—namely, safer adjustment to environment. We may express this motor aspect of the judgment by saying that it introduces *more advantageous control.*

It must have appeared already that judging is a process of ascribing *objective validity to our mental states.* Its function is to secure a *harmonious adjustment between mind and fact, man and his environment.* It must be noted that in such judging activities we first of all standardize the world for ourselves; secondly, we attempt to standardize it for all other sane minds; for we are never content until our judgments find approval in the society about us. Either we are wrong or the social environment is wrong until a nice adjustment is secured. Thus we modify our judgments as they are brought into contact with those of others, and we strive valiantly and strenuously to bring others to acquiesce in our judgments. In the interplay of opinion the types of standard judgments called principles emerge. It must also be noticed that we modify our judgments

to bring them into better harmony with fact. It is out of such modification that modern scientific judgments have developed. But it is equally interesting to remark that certain types of subjective judgment are maintained in face of the stubborn resistance of the fact world. Our æsthetic, moral, and religious judgments maintain a bold insistence upon *what ought to be* in face of the most stubborn reiteration by hard facts *of what is.* But even these higher spiritual judgments have a solid basis in experience, which is continually invested by our minds with a significance that reaches beyond itself. For out of experience are formed all the higher ideals toward which we direct the ongoings of evolution.

41. Stages of Judgment.—The evolution of the judging function progresses through certain fairly well-defined stages. Beginning with the infant's earliest mental processes, in which part of his experience is referred to an outer source—the eternal world about him—and the other part to his own bodily and mental changes, there is gradual progress until the nice discriminations of mature life are made. The first stage is that in which *sensations coming from the outside are distinguished from states of body and mind.* Here begins that great division of the world into environment and self, which is ever after to furnish us with our most important distinction. Another stage that comes scarcely later is that in which *the social world of self and other selves* is discriminated. The importance of this lies in the fact that ethical values take their rise out of it. Still another stage in the development of judgment is reached when the child begins *to put its experiences into*

classes on the ground of their similarities and differences. This is essentially the concept-forming tendency. The world around the child is divided up into things that are alike and unlike. As the classifications which lead to the concepts get more accurate the pupil approaches scientific knowledge. A still more advanced stage is reached when the mind begins *to detect relationships between classes of things.* The judgments growing out of this are often expressed in verbal form and are then called sentences or propositions. Finally, the maturing mind, having a stock of judgments in which concepts are related, *connects these judgments by noting their common agreement in some particular, and we have a complex type of judgment technically called by the logician syllogistic reasoning.*

Aside from the main stages of judgment here mentioned, we may notice one or two others, which are really forms of the above. Foremost among the minor forms alluded to is what we may call the *abstracting judgment.* This is a kind of concept-forming. After a classification has been made and the appropriate concept developed, attention is directed to determining just those qualities that are necessary to the concept. Comparison is made with other concepts possessing the same attributes, and the thought of the attribute apart from any particular concept in which it is found emerges. Thus we get abstract concepts through the focussing of attention upon qualities or attributes, and through the activity of the mind's judging function, which tends to assert as real or existent anything which comes repeatedly under attention. Logicians have usually dis-

tinguished the *analytic* from the *synthetic* judgment. The former merely brings to the attention in a definite way some aspect or quality of a thing or concept that is really implied in our thought of it. The synthetic judgment adds something to the concept which was not necessarily implied in our thought of it. If we say, Wood has weight, we are stating an analytic judgment; for weight is necessarily implied in the thought of wood as a material substance. On the other hand, when we say, Wood is combustible, we state a synthetic judgment; for the concept of wood can be thought without implying the attribute of combustibility. This distinction is a logical rather than a psychological one; for psychologically every judgment is the result of both an analytic and a synthetic mental process. Nor can we defend the distinction with very great assurance on even logical grounds; for the difference between what is implied in a concept and what is added to it is nine times in ten far-fetched and artificial.

What has just been said must not be construed as meaning that the several "stages" of judgment noticed succeed one another in a rigid series. The order of their unfolding may differ somewhat with individuals. One stage may overlap another. But in a general way they tend to develop gradually in about the order named. We may set this order down in the following table:

First phase of judgment: *Distinction between self and environment.*

Second phase of judgment: *Distinction between self and other selves.*

Third phase of judgment: *Distinction between kinds of things.*
Fourth phase of judgment: *Relating kinds of experiences.*
Fifth phase of judgment: *Relating organized experiences.*

42. Types of Judgment.—There are three distinct types into which a judgment may fall, viz., *categorical, hypothetical,* and *disjunctive.* When expressed in propositional form they are illustrated as follows: *categorical,* Deer are cloven-hoofed quadrupeds; *hypothetical,* If a mammal chews its cud it has a compound stomach; *disjunctive,* This specimen is either a moth or a butterfly. A brief examination of these types of judgment will show us that they are adapted to somewhat different purposes and represent rather different mental attitudes. The categorical judgment is essentially a classificatory one. When the mind is absolutely certain of a changeless relation between two concepts—or when it deludes itself into believing that this certainty has been reached—the categorical judgment is adapted to its needs. Now absolute certainty of knowledge results either from ignorance or from old-fogyism. A larger interpretation of experience requires us to allow for constant development in our concepts, in the relations established between them, and indeed in all the several forms of judgment. Hence in strictness only the child and the old man can use the categorical judgment with its full weight. Most of us use it with a real or implied mental reservation to the effect that the truth we assert in it is only relative, and is hence subject to correction from larger experience.

The hypothetical judgment is peculiarly adapted to stating the conditions under which an occurrence may take place. Hence it is the form for recording causal

conditions; and naturally it represents the various aspects of experience as mechanically related. It is, in consequence, the natural form for scientific judgments. We might almost say that all exact science is really an assertion of "ifs." For back of our science lies the fundamental metaphysical hypothesis as to the real being or nature of the world. The actual reasoning of science is perhaps a little dubious from the logical standpoint. When we lend the assent of reason to a great scientific hypothesis, such, let us say, as the nebular hypothesis, we argue as follows: If the nebular hypothesis were true, the solar system would behave as it does. Then we proceed to argue *back to the truth of the nebular hypothesis as though we had proved it the sole condition of the solar system's moving as it does.* In other words, the hypothetical judgment in which science must state its claims is not final.

Last of all comes the judgment of true metaphysical ground or reason, the disjunctive judgment. It assumes that a situation has been worked out until representable in contradictory alternatives. To one of these the mind assents on account of the nature of the subject under judgment, and this involves the rejection of the other alternative. This is in the strictest sense the judgment of rationality. For it implies that the ground has been thoroughly traversed, that the problem has been viewed in all its relations, and that the basis of explanation bearing upon its solution has been reduced to contradictory alternatives, with one of which we identify the problem in question, and from the other of which we thereby reject it.

The three types of judgment here referred to develop, roughly speaking, in the order set down. The child's judgments tend to be categorical, the judgments of the practical scientist hypothetical, while those of the philosopher who tries to explain the world on a rational ground are thrown into the disjunctive form. And in a general way all minds pass through phases of development that may be roughly analyzed as naïvely dogmatic, as scientific, and finally as philosophical.

43. The Traditional Interpretation of the Judging Function by Logic.—If one were to consult the older text-books of logic he would notice that the view of the judgment there taken narrows it down very greatly in comparison with that just given. This is to be accounted for by the fact that the science used to be approached more from the language stand-point than at present. Hence the judgment was not regarded so much as a *psychological product*, but rather as a *language form*. In this way it was erroneously identified with the proposition or was at least limited to *such mental activity as might take propositional form*. This narrow view was further justified by the old faculty notion of the mind, according to which the mind was made of isolated powers, each definitely limited to its own distinctive work. This traditional interpretation has had to give way, like many another good and hoary superstition, before the evolutionary conception of mental functions. The view explained above and subscribed to in this book regards judgment as the *unit of thought-activity*, having manifold forms and applied to innumerable uses, but always as a mental function the same.

From babyhood to maturity it develops mightily; but never does it cease to be judgment and become some new phenomenon. The first awareness of the infant is a judgment-form; the final word of the philosopher is one also, only more elaborate and complex. In fact, it may be said that the knowing aspect of the mind's activity is always some form of judging. Intellect, understanding, classification, perception, conception, judgment (in the old, narrow sense), reasoning—these are all but so many names for forms of judging. And since our logical interests centre entirely in making these manifold intellectual acts of judging true and fruitful instead of false and abortive, we are correct in pronouncing logic to be *the science of valid judging.*

44. Implicit and Explicit Judgment.—We have just said that judging is represented in all forms, from the naïve perceptions of the "baby new to earth and sky" to the profound and complex theses of the philosopher. Hence one useful distinction is that between *naïve* and *reflective* judgments. The former characterize the inexperience of child life, and likewise the inexperience of the sophisticated man when he deals with a novel problem. The naïve judgment issues out of our inexperience. Lacking an apperceptive ground of organized knowledge by which to interpret the novel challenge to the attention, the child and the adult who has strayed away from the accustomed haunts of his intellect alike assert a judgment which may be quite foreign to the real nature of the fact. The reflective judgment, as the name indicates, refers the matter awaiting judgment to a rich content of organized experi-

ence, and is so much the surer to reach the truth. There is another useful distinction, that between the *implicit* and the *explicit* judgment. By far the larger number of our judgments never get conscious language-form. We may unconsciously think them in silent words after we have a vocabulary at our command; but we rarely speak them aloud unless we wish them to secure the assent of others. Infants who cannot as yet speak make implicit judgments of a vague kind. Our explicit judgments assume various forms, the most common of which are the *name,* the *logical sentence* or *proposition,* and the several forms of *syllogistic reasoning.*

45. THE PROPOSITION.—So far as the interests of logic are concerned, the proposition or logical sentence is one of the most important of the several forms of explicit judgment. This is the case because it is the vehicle through which we give expression to our inductively generalized experience, throwing it open to acceptance or challenge. The importance of this parade of the judgment before others is great. It enables us to supplement our experience by that of others, and in this way we may correct our individual errors and strengthen our feeling of certitude or our cognitive faith. Indeed, knowledge is far more a social product than we suspect. Until a judgment meets approval it is little more than a guess at the truth. Let it be approved by others and it becomes the truth for many, as is shown by false doctrines and beliefs that gain popular currency by mere numbers. The proposition is therefore of vast help in developing bodies of truth. It allows your judgment

and mine to come into the arena of intellectual conflict and there to struggle for existence with abetting and conflicting judgments. It is from such a struggle that the great common beliefs emerge.

With beings whose business it is to use language there must be a great value in having the judgment put into language. The proposition results in clear thinking such as could not be accomplished without it. Any one who has had the experience of seeking just the right turn of a phrase to express his thought knows how his thinking is cleared up when the desired language occurs to the mind. The psychological observer can prove this statement in every examination hall. Brows are contracted, faces wear a baffled look, the stamp of failure is in evidence, until suddenly the right expression is found. Thus the proposition is a constant aid to the judging activity of the mind.

46. Form of the Proposition.—The proposition in its simplest form is made up of two terms united by a form of the copulative verb *to be.* This is also the language unit and standard of the judgment. Though the majority of our judgments are expressed in some other way, it is thought that these simple elements may be detected in all of them. Of course the negative judgment must include besides some form of negation. It is a good practice for students of logic to try to detect these simple elements in more complexly expressed propositions. In this way a habit of seeing the logical value of involved discourse may be acquired. It is well to have in mind certain standard language-forms to which the four types of judgment may be reduced.

These four types are the two judgments that include all the things referred to by the first or subject term, and are hence called *universal*, and the other two that include only part of the things to which the first term refers, and are therefore *particular*. One of each of the above is *positive* or *affirmative*, the other *negative*. Hence we have a *universal affirmative*, a *universal negative*, a *particular affirmative*, and a *particular negative*, designated for brevity's sake by the letters A, E, I, O, respectively. The first and third letters are the first two vowels of the Latin *affirmo*, the second and fourth from *nego*. Standard language-forms for these propositions may be stated as follows:

(A) *All . . . are . . .*
(*All caterpillars are insects.*)

(E) *No . . . are . . .*
(*No caterpillars are true worms.*)

(I) *Some . . . are . . .*
(*Some caterpillars are green.*)

(O) *Some . . . are not . .*
(*Some caterpillars are not green.*)

The student should be careful to note the equivocal character of the commonly used negative form, All . . . are not. It may be used to express an E proposition, *e. g.*, All caterpillars are not true worms (meaning, No caterpillars are true worms). It may also convey a particular negative judgment and imply a particular affirmative one; as in the proposition, All students are not trustworthy. Here the statement means, Some students are not trustworthy. It also implies, Some students are trustworthy. The proposition, All students are not trustworthy, would be better in the form, Not all students are trustworthy.

On account of its ambiguity the form, All . . . are not, is to be carefully avoided.

47. Tables Showing the Classification of Propositions Expressing Categorical Judgments.—The following tables will help the student to see at a glance and to remember the forms and the relations of the several propositions which are employed to give expression to categorical judgments:

Kinds of propositions	Quality	Affirmative: assert the predicate of the subject.
		Negative: deny the predicate of the subject.
	Quantity	Universal: assert or deny the predicate of all of the subject.
		Particular: assert or deny the predicate of some of the subject.

Symbols for the four propositions	Universal	Affirmative	A (*A*ffirmo).
		Negative	E (n*E*go).
	Particular	Affirmative	I (aff*I*rmo).
		Negative	O (neg*O*).

48. Language Signs of Quantity and Quality.—It is well that the student should keep in mind the most common language-forms that are used to announce the character of a categorical proposition; for by so doing he will have the means for a quick inspection and determination of the logical value of the sentences he meets with in reading and speech. *Any word or word-group that plainly indicates that all of the subject (i. e., the full denotation of the term) is referred to in the proposition is a sign of universal quantity.* The most common word is *all* (meaning *each and every, i. e., used distributively, not collectively*); other words are *every, each, any. Any*

word that plainly indicates that only some part (*i. e., less than the full denotation*) *of the term used as subject is referred to in the proposition is a sign of particular quantity.* The most common word is *some;* other words are *certain, a few, many, several, a number of,* and *most.* The most common sign of negative quality is the adverb *not.* Oftentimes, however, the negative is attracted from the copula or predicate to the subject, where it usually has the form of a negative adjective or pronoun, such as *no, not any, none, not one,* etc.

49. COMMON VARIATIONS FROM THE LOGICAL FORM. —Certain typical variations from the strict logical form should be carefully noted at this point and their interpretation remembered. This is very important because these variations are constantly met with, and prove very troublesome to the student who is not thoroughly familiar with their meaning.

Singular propositions are those whose subject refers to an individual person or thing. They are regarded as being universal because the whole subject is necessarily referred to.

Indefinite (*or indesignate*) *propositions are such as have no sign of quantity* (neither *all, some,* nor synonyms). They are not in a form for logical treatment unless, as often happens, the quantity intended is perfectly obvious. When this is the case all that is necessary is to insert the proper sign of quantity.

Examples of the above variations are: (singular) *Socrates was a great philosopher;* (indefinite) *Known planets are members of our solar system.* The last sentence evidently means to say that ALL *known planets are*

members of our solar system; hence we insert the word *all* and have a universal proposition.

Exclusive propositions are those which are introduced by such words as only, alone, none but, which are intended to indicate that the predicate-concept lies within the denotation of the subject-concept. That is to say, these propositions really show that the subject-matter named by the predicate is part of but not necessarily all of the subject-matter referred to by the subject-term. For example, when the assertion is made, *Rational beings alone are morally responsible, Only rational beings are morally responsible,* or *None but rational beings are morally responsible,* the real meaning is that morally responsible beings are found only among rational beings.

The novice in logic must avoid the error of thinking that such a proposition implies that the denotation of the concept, rational beings, is the same as that of morally responsible beings. We make no assertion to that effect. Rational beings may include many more things than morally responsible beings for aught our proposition tells us. But we do know from the proposition that morally responsible beings can include no more things than rational beings. The best way to treat the exclusive proposition is to change it to an A proposition, which will convey the same meaning. This is done by observing the following simple rule: *To express an exclusive proposition in the A form, interchange the terms and write the new subject-term (the predicate-term of the original exclusive proposition) with the word, all, before it.* Thus, from *Rational beings alone are morally responsible,* we get, *All morally responsible beings are rational,* and this is *all* that we meant to say and *no more.* Pupils find exclusive propositions hard to understand, and hence need to make and reduce them to an A form until they have a thorough mastery of them.

Duplex propositions are those which state one meaning and plainly imply another judgment also. They may be distinguished into *partitive* and *exceptive.* *Partitive*

propositions make an assertion respecting part of a class and imply a judgment respecting another part. Ambiguity in the meaning of *All . . . not,*[1] *some, a few, part of,* etc., usually indicated by a stress of voice on these words, is responsible for this kind of duplex meaning. Under such circumstances there is a plain implication of another judgment besides the one verbally expressed, having *the opposite quality.* For example, Some *are on time* evidently implies that some (*others*) *are not on time;* All *are not on time* (which means *Not all are on time*) plainly implies that some *are on time;* A few *of the students in logic were conditioned* implies that some (*many*) *of the students in logic were not conditioned.*

Exceptive propositions assert a predicate of all of a subject with the exception of certain individuals. If the latter are definitely designated the proposition practically states two judgments, one in which the predicate is attributed to a certain part of the class and another in which the contradictory of the predicate is by implication attributed to the excepted individuals. The most common introductory words to exceptive propositions are *All but, All except, All save,* etc. The following proposition illustrates this sort of predication: *All the planets, with the exception of Venus and Mercury, are more distant from the sun than the earth is.* The two judgments involved are: *Mars, Jupiter, Saturn, Uranus, and Neptune are more distant from the sun than the earth is;* and *Venus and Mercury are not more distant from the sun than the earth is.*

[1] *Cf.* p. 77.

50. Grammatical Order and Logical Meaning. —In order to secure emphasis, for the sake of variety, or as a result of careless construction, the writer or speaker may throw his sentence into an irregular order that may prove a source of confusion to the logician. Under such circumstances the sentence must be recast before it can be a logical proposition. The logician must remember his propositional unit, consisting of subject and predicate terms, united by the copula, and must determine these values from the irregularly constructed sentence before him. If he cannot determine these values definitely, the sentence cannot be used in logic until the original framer of it succeeds in conveying his meaning more definitely. It is evident that we have here a problem that is partly grammatical, partly logical. The grammatical aspect lies in making sure what kind of irregular construction is before us and in seeing that it is not a violation of the rules of syntax; the logical aspect consists in establishing the meaning that the irregular construction was meant to convey.

Inverted order occurs where the predicate has the position usually occupied by the subject and vice versa. It is very common in poetical and oratorical diction. Before using such a sentence in logic, it must be put into the normal order by putting each term in the place assigned to it in a proposition, that is, the subject-term first, then the copula, and last the predicate-term. For example, the inverted sentence, "*Great is Diana of the Ephesians,*" must be reconstructed in the usual order, *Diana of the Ephesians is great,* when we at once recognize it as a singular proposition having universal value. *Short are*

the days in the spring-time of life becomes logically (*All*) *the days in the spring-time of life are short.*

A transposed relative clause consists in separating an adjective clause from the word it is meant to modify and placing it near another word which its adjective function in the sentence would permit it to modify. It is a case of more serious trouble than the inverted order, and not infrequently results from careless style rather than from design. When very misleading it may occasion a species of the fallacy of amphiboly. The logician's task in this situation is to determine where the adjective phrase properly belongs and to put it there before proceding further. Example: *No one is capable of command who cannot rule himself.* This becomes, when properly put, *No one who cannot rule himself is capable of command.* Here the subject-term in full is, *no one who cannot rule himself;* the predicate term, *capable of command. All is not gold that glitters* should be changed to *All that glitters is not gold;* or better, to conform strictly to the propositional standard (*see p.* 77), *Not all that glitters is gold*, or, *Some things that glitter are not gold* (an O meaning is what the author of the proverb intends to convey). Here the subject-term is, *all that glitters* (*not all that glitters* or *some things that glitter*); the predicate-term, *gold.*

Compound sentences can be regarded as propositions only after they are broken up into several separate assertions. For example, *The soldiers are obedient to command, and the general is a master of strategy,* consists of two separate propositions which must be treated severally in logic, the one being, *The soldiers are obedient to*

command, the other, *The general is a master of strategy.* But it may often happen that a compound subject really means a unit in the writer's or speaker's thought; as in the sentence, *The boys and girls in Miss B's school are unusually attentive to their lessons,* the evident meaning of the two words *boys* and *girls* is to call to mind not difference in sex but the unity of activity implied in being pupils of the school in question. Under such circumstances, where the two or more terms evidently are used as substitutes for a single generic term like *pupils of Miss B's school,* they may be used together in a true proposition. The same thing is true of a compound predicate, though the instances are probably rarer. I may say with logical propriety, *The boys and girls* (*pupils*) *of Miss B's school are unusually attentive and willing,* if I mean the predicate-terms *attentive* and *willing* to be used together as signifying the generic concept *well trained, moral, industrious,* or something similar.

This may all be summed up by cautioning the student to observe the following rule in interpreting sentences: *First, put the sentence into normal order; second, determine the quantity and quality that are intended and write down the symbol of the proposition; third, be sure to see in the proposition the entire subject and predicate terms and the copula; fourth, be sure to see in the sentence the standard propositional form, even though you do not reduce it to that form; fifth, work these several steps out carefully at first in actual practice upon examples, until there is developed a habit of feeling the logical value of sentences; sixth, even after long practice, in case of con-*

fusion, put the sentence through all the steps indicated if it can be done; if not, reject it as not capable of logical interpretation.

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REVIEW QUESTIONS

1. *In view of the importance of judging, how may we define logic?*
2. *Name several early mental processes that are to be explained as forms of judging.*
3. *Show how judging is involved in the child's first consciousness; in the formation of concepts; in relating concepts.*
4. *How may judging be defined?*
5. *Discuss how we modify our judgments to secure social approval for them and to make them harmonize with fact.*
6. *Name some instances of judgments expressing what ought to be rather than what is.*
7. *What relation have judgments respecting what ought to be to social and moral evolution?*
8. *Name and discuss the stages of judging.*
9. *What is the abstracting judgment?*
10. *Distinguish between the analytic and the synthetic types of judgment.*
11. *What do you understand by the categorical judgment, and to what use is it peculiarly adapted?*
12. *What do you understand by the hypothetical judgment, and what is its special use?*
13. *What do you understand by the disjunctive judgment, and what is its special use?*

14. *Contrast the older and the newer (evolutionary) conceptions of judgment.*

15. *Which of the above-mentioned conceptions is the more inclusive? Explain why.*

16. *Name several different forms of judging.*

17. *Contrast naïve with reflective judging. Cite instances.*

18. *Contrast implicit with explicit judgments. Cite instances.*

19. *What are the most common forms of explicit judgment?*

20. *Give reasons for the logical importance of the proposition.*

21. *What does language do for the judgment?*

22. *What elements may be detected in the proposition?*

23. *Name and give the symbols of the four kinds of propositions.*

24. *Memorize and give the four standard logical forms for the four propositions.*

25. *Explain the confusion to which the form, All . . . are not, is liable.*

26. *What letters are used to symbolize the four propositions?*

27. *What are the most common verbal signs of quantity and quality?*

28. *What is a singular proposition and how is it interpreted in logic?*

29. *How are indefinite propositions to be treated?*

30. *What is the exclusive proposition equivalent to and how do you determine its equivalent in a given case?*

31. *What are duplex propositions and what two kinds may be distinguished?*

32. *What does a partitive duplex proposition imply?*

33. *What is implied in an exceptive duplex proposition?*

34. *What are the most common sources of unusual order in sentences?*

35. *What is the logician's duty toward such sentences?*

36. *What is to be done with a sentence in the inverted order?*

37. *How is a transposed relative clause to be construed in logic?*

38. *How are compound sentences and compound terms to be dealt with by the logician, and what is the exception to the rule respecting them?*

39. *What is the rule for interpreting sentences? for expressing our meaning in logical form?*

EXERCISES ON CHAPTER VII

1. Write out two judgments in the categorical form.
2. Write out two judgments in the hypothetical form.
3. Write out two judgments in the disjunctive form.
4. Combine the following concepts so as to make as many judgments as you can, some being affirmative, some negative, some particular, and some universal; then express your judgments in as simple propositions as you can: *day, yellow, flower, fine, girl, slow, school, logic, large, pretty, easy, houses on this street, students of logic, beautiful in design, more accurate reasoners.*
5. Find the logical elements in the following complex propositions, and restate the several judgments in the standard logical forms (awkward English will often result, but this does not matter from our present point of view, although no violations of grammatical rules are permissible):

(*a*) *Happy is the man that findeth wisdom.*
(*b*) *It is immoral to lie with malicious purpose.*
(*c*) *All the students who elect this course have in view a business career.*
(*d*) *It is reported that Japan has declared war against Russia.*
(*e*) *A strong instinct for individual freedom combined with profound respect for legalized authority constitutes the bulwark of American government.*
(*f*) *Taxation without representation is tyranny.*
(*g*) *All men are created free and equal.*

6. Express in the standard logical forms all that you are sure is true of the following concepts, using them just as they are arranged here, but supplying such additional words as are necessary to put the truth into the proper form:

(*a*) *dogs* —— —— *gentle.*
(*b*) *insects* —— —— *six-legged.*
(*c*) *caterpillars* —— —— *centipedes.*
(*d*) *horses* —— —— *capable of domestication.*
(*e*) *wolves* —— —— *faithful animals.*
(*f*) *days* —— —— *pleasant.*

7. Correct the logical form of the following propositions so as to avoid the colloquial error which they illustrate, and render them true to fact as your experience gives it to you:

(*a*) *All dogs are not hounds.*
(*b*) *All men are not liars.*
(*c*) *All sheep are not white.*
(*d*) *All studies are not tedious.*
(*e*) *All days are not clear.*
(*f*) *All books are not novels.*
(*g*) *All girls are not logical.*
(*h*) *All students are not industrious.*

8. Put the following sentences into the standard logical form appropriate to express the truth about the subject-matter used:

(*a*) *Men are not capable of unremitting toil.*
(*b*) *None of the expected guests are here.*
(*c*) *Men are not infallible.*
(*d*) *Malicious liars are not trustworthy.*
(*e*) *Reptiles are not warm-blooded.*
(*f*) *All moral persons are not persons of good judgment.*
(*g*) *Spiders are not insects.*
(*h*) *All cloudy days are not rainy days.*
(*i*) *All sunny days are not windy days.*

9. Make your own example of each of the four categorical propositions, and employ with each proposition all of the language signs of quantity and quality given in the text that are appropriate to the proposition in question.

10. Determine which of the following propositions are singular and which indefinite; be prepared to explain why each of the former is universal, and attach (if possible) the proper sign of quantity to each of the latter:

(*a*) *Louis XVI was not a wise king.*

(*b*) *Plato is the author of a dialogue which deals with educational theory.*

(*c*) *A good man is assured of the approval of his own conscience.*

(*d*) *Fixed stars are suns of other solar systems.*

(*e*) *True worms are not destined to the metamorphosis of caterpillars.*

(*f*) *Eels are less highly organized than snakes.*

(*g*) *An eel is a species of fish.*

(*h*) *The rattlesnake is a member of the family of pit-vipers.*

(*i*) *Abraham Lincoln was an eminent statesman.*

11. Recast the following exclusive propositions in the A form, and be prepared to explain how the A gives the same meaning; then make four exclusive propositions of your own and state them in the A form; finally, make four original A propositions and recast them as exclusive propositions:

(*a*) *None but good logicians are able to detect the sophistry of a skilful criminal lawyer.*

(*b*) *Only those who avoid temptation are sure of a moral life.*

(*c*) *The virtuous alone are the truly happy.*

(*d*) *None save those who strive for wisdom are attaining the full destiny of rational beings.*

(*e*) *None but the brave deserve the fair.*

12. State each of the following duplex sentences in two propositions so as to make explicit the implied meaning, first determining which kind of duplex proposition each is:

(*a*) *Some of this fruit is decayed.*
(*b*) *A few of the audience applauded.*
(*c*) *A part of these books is soiled.*
(*d*) *Some of the wedding guests have not arrived.*
(*e*) *All but Miss A. and Mr. B. are going.*
(*f*) *All except five were rescued from the wrecked ship.*
(*g*) *The majority favor the proposed amendment.*
(*h*) *All save three voted for him.*

13. Determine what type of irregularity the following sentences represent, then arrange them in good propositional form:

(*a*) *High are the walls of Babylon.*
(*b*) *All are not trustworthy who make a parade of virtue.*
(*c*) *Straight is the gate and narrow is the way.*

(*d*) *All is not misfortune that seems at the moment unbearable.*

(*e*) *Many are those who seek the favor of kings.*

(*f*) *She was with him whom I sought.*

(*g*) *Mighty are the oaks of Epping Forest.*

CHAPTER VIII.—OPPOSITION AND TRANSFORMATION OF CATEGORICAL PROPOSITIONS

51. Knowledge Value of the Four Categorical Judgments.—At this point we must determine exactly what amount of knowledge respecting each concept in its relation to the other the four categorical judgments convey. It must be borne in mind that the judgment states our experience respecting the subject-matter under consideration, and is conveyed in appropriate propositional form with the purpose of gaining the assent of others. We are supposing here that we strive to render judgments which are true to our actual experience, and that the several propositions in which we couch those judgments are so framed as to render our full knowledge in standard logical form.

We shall consider first the knowledge that we feel warranted in asserting in a judgment which we state as an A proposition. Take, for instance, the proposition, *All amphibians in the adult form breathe by lungs.* An instant's thought shows that an assertion is here made respecting each and every individual belonging to the class indicated by the term *amphibians in the adult form;* the assertion, namely, that they breathe by lungs. In other words, the full denotation of the subject-term is meant. On the other hand, no assertion is intended

respecting each and every individual belonging to the class indicated by the term *breathers by lungs.* So far as the judgment under consideration is concerned, our knowledge does not allow us to predicate anything universally respecting the predicate-term. This may be summed up by saying that the A proposition asserts the predicate-term respecting each and every individual (the full denotation) of the subject-term, whereas the subject-term does not convey any knowledge respecting each and every individual (the full denotation) of the predicate-term.

Next let us consider the E proposition. As an instance let us take the proposition, *No bats are birds.* In this proposition we give voice to a judgment that gives negative knowledge regarding each and every individual belonging to the class signified by the term *bat,* for it tells us one thing that each and every bat is *not,* viz., a bird. The proposition also gives negative knowledge respecting each and every individual referred to by the predicate-term *bird,* for in denying that bats are birds we also necessarily preclude the possibility of maintaining that birds are bats. That is to say, by severing all possibility of relationship between the two concepts represented by the terms *bat* and *bird,* negative knowledge is conveyed respecting each and every individual referred to by either concept.

As an example of the I proposition may be taken the judgment that we feel warranted in asserting in the sentence, *Some dogs are hounds.* It is evident that nothing is asserted about each and every individual included in the concept referred to by the term *dog,* for we qualify

our affirmation by the word *some,* with the special intention of referring only to an indefinite part of the class dogs. We only mean to say what some dogs are, viz., hounds. And in this proposition we do not assert anything regarding each and every individual referred to by the term *hound.* For although it chances that our experience tells us that all hounds (each and every hound) are dogs, we have to frame another proposition to express such a judgment. Another example will better illustrate how erroneous it would be to maintain that the predicate-term is employed universally in the I proposition. Take the sentence, *Some pupils in this class are bright girls.* It would certainly do violence to the facts of experience to claim that here some one assertion is made regarding each and every individual symbolized by the predicate-term *bright girl;* for as the only concept which is related to the predicate-term in this proposition is the subject-term *pupils in this class,* we should fall into the absurdity of maintaining the judgment that *each and every bright girl is a pupil in this class.*

The following proposition illustrates the O form of judgment: *Some educated men are not moral.* Here we are not asserting any one thing respecting each and every individual of the class referred to by the term *educated men,* for we qualify that term by the indefinitely limiting word *some.* Hence the subject-term is not taken universally. The predicate-term, on the other hand, has something told respecting it that refers to each and every individual falling within the class denoted by it. One item of (negative) information is conveyed

to the mind respecting every moral being in existence, viz., that he is not one of the group of *some educated men* indicated by the subject-term. This is summed up by saying that the subject-term of the O proposition does not furnish the mind with any one item of knowledge regarding its full denotation, whereas the predicate-term of the O does supply such knowledge.

The whole of this subject of the knowledge value of the several categorical propositions may be briefly stated as follows: Our problem is to tell how complete is the knowledge vouchsafed us in regard to each term. The universal propositions A and E are so called because their subject-terms are used in full denotation, that is, are meant to refer to each and every individual belonging to the class symbolized by the term. In contradistinction to them, the I and O propositions are called particular because their subject-terms are not used in full denotation, as is indicated by the word *some*, or its equivalent, prefixed to them. Hence it is clear that the subject-terms of universal propositions are meant in full denotation, while those of particular propositions are not so meant. But so far nothing is shown respecting the predicate-terms. A moment's thought shows us that the negative propositions E and O indicate a complete severance of relation between the subject and predicate terms; hence they must necessarily furnish us with knowledge of a negative character affecting every individual falling within the denotation of the predicate-terms; for we know in these two propositions at least *one* thing that the individuals denoted by the predicate-terms are *not*, viz., the individuals referred to or such

part as is meant by the subject-term. The predicate-terms of the affirmative propositions A and I make no such complete contribution to our knowledge, for they are left entirely indefinite, so that from the propositions as they stand we cannot tell whether all or only a part of the denotation is intended.

The subject that has just been discussed is technically referred to in logic as *distribution of terms, which means the character of their denotation, i. e., whether all the individuals belonging to the concept, or only some indefinite part of them, are meant.* Distribution is an abstract noun; corresponding to it are the two adjective forms *distributed* and *undistributed. A term is said to be distributed when used in a proposition to refer to the full denotation of (each and every individual in) the class which it symbolizes. A term is said to be undistributed if used in a proposition to refer to only some indefinite part of its full denotation, i. e., to some only of all the individuals of the class which it symbolizes.*

The results of the above discussion may be expressed in this table:

PROPOSITION	SUBJECT-TERM	PREDICATE-TERM
A	D (+)	U (−)
E	D (+)	D (+)
I	U (−)	U (−)
O	U (−)	D (+)

D or + means *distributed.*
U or − means *undistributed.*

Of course only two distributions are possible in a given proposition. Remembering this, we can also summarize as follows: E distributes *both;* I distributes

neither; A distributes the *subject-term;* O distributes the *predicate-term.* We also notice that a proposition must be *universal to distribute its subject-term* and *negative to distribute its predicate term.* Hence the proposition E, which is *both universal and negative, distributes both terms,* while the proposition I, which is *neither universal nor negative, distributes neither term.*

52. Conditioning Relationships between Categorical Judgments having the Same Subject-matter.—A little examination will discover to us that there are certain limiting or conditioning relationships between the several propositions providing we attempt to assert them about the same subject-matter. An A and an E proposition are inconsistent with each other. Thus the proposition, *All citizens are voters,* is absolutely inconsistent with the proposition, *No citizens are voters.* Manifestly, both cannot be true. At the same time it may be the case that neither an A nor an E expresses our true experience, for the facts may lie somewhere between *all* and *none.* For instance, it would be untrue to experience to maintain either that *all sheep are white* or that *no sheep are white,* for as respects this subject-matter experience leads to particular, not universal, judgments, the truth being that *some sheep are* and *some sheep are not white.* Logic calls A and E *contraries, meaning by this term that both cannot be true of the same subject-matter, but that neither may be true.*

An A in relation to an O, and an E in relation to an I, or vice versa, are also inconsistent. Thus, if the judgment expressed in the proposition, *All men are desirous of happiness,* is true, it is impossible that the proposition,

Some men are not desirous of happiness, could also be true, and vice versa. And if the proposition, *No spiders are insects*, is true, it is impossible that the proposition, *Some spiders are insects*, could likewise be true, and vice versa. But the inconsistency between these last considered propositions is different from and in a way more complete than that between A and E; for both A and O or E and I cannot be false. For the falsity of an A, such as *All men are sincere*, proves the truth of the O, having the same subject-matter, viz., *Some men are not sincere*. So the falsity of an I, *Some crows are white*, forces us to assent to the E with the same subject-matter, *No crows are white*. The logician expresses the relation between A and O, E and I, and vice versa, by calling them *contradictories, which means that not only is it impossible for both to be true, but that also one must be true, the truth of one proving the falsity of the other, and the falsity of one proving the truth of the other.*

The relation between any universal and the particular with the same subject-matter and quality is such that the universal being true the particular must also be true. But the truth of a particular does not compel the truth of the corresponding universal. Thus, if it is true that *all men are fallible*, it must be true that *some men (at least) are fallible*. But the fact that *some dogs are black* does not involve one in the conclusion that *all dogs are black*. The principle underlying our reasoning here is the so-called *dictum of Aristotle* that *what is true of the whole is true of the part*. The logician calls the relation between an A and an I, or vice versa, and an E and an O, or vice versa, having the

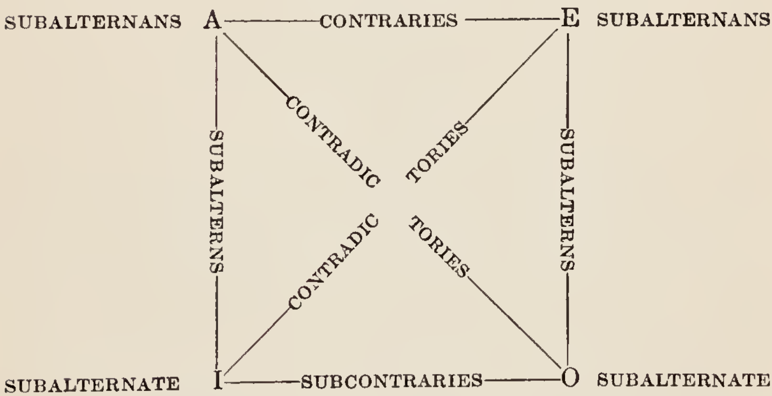
same subject-matter, that of *subalterns.* The A and E are each called a *subalternans* (the Latin present active participle, meaning to *render of lower rank*), the I and the O, respectively, their *subalternates* (the Latin past (passive) participle, meaning *made of lower rank*).

The relation between the two particulars dealing with the same subject-matter is such that, while they may both be true, and in application to much of our experience are both true, yet at the same time the truth of either militates somewhat against the probability of the truth of the other. Our experience must often be embodied in the form of qualified predication. Thus our experience of the subject-matter referred to by the term *animal* forces us to make the qualified predications, *Some animals are warm-blooded* and *Some animals are not warm-blooded.* In this instance both particulars are true and must be entertained as expressing the judgments which experience forces upon us. But, on the other hand, there is the instance of the knowledge which we are warranted in asserting as the result of a developing inductive inquiry, which has revealed no exceptions up to the moment of judging, but in which we are aware that not all cases have been examined. All that a careful investigator is permitted to assert is a particular proposition which recognizes the full field of possible knowledge as unexplored, and hence unknown. For example, let us suppose that a student of biology dissects a score of tadpoles and finds that they all breathe by gills. Not having examined all tadpoles, and having consulted no authority, all that he could safely affirm is, *Some tadpoles breathe by gills; some tad-*

poles do not breathe by gills being a judgment that still lies within the range of possible truth. But as the investigator proceeds to dissect more and more tadpoles the likelihood of the truth of the O continually diminishes. It is thus seen that the truth of either I or O makes somewhat against the chance of the truth of an O or an I, with the same subject-matter. This slightly opposing relationship is expressed by calling them *subcontraries.*

The technical name for the relationships discussed here is *opposition, by which is meant the conditioning influence of any one of the four categorical judgments upon others having the same subject-matter.*

SQUARE OF OPPOSITION
(ARISTOTLE'S SQUARE)



53. Square of Opposition.—It is a long-standing tradition of logic texts to express these relations graphically in a diagram to which has been given the name of the Square of Opposition, or Aristotle's Square. It furnishes a convenient means of committing these extremely important relations to memory.

54. Summary of Relations.—The following statements summarize the relations of propositions:

1. Both contraries cannot be true, and both may be false.
2. One of each pair of contradictories must be true, and one false.
3. Both subcontraries may be true, and one only can be false.
4. Subalterns: the truth of the subalternans involves the truth of its subalternate, but the truth of the subalternate leaves the truth of its subalternans undetermined; the falsity of a subalternate involves the falsity of its subalternans, but the falsity of a subalternans leaves the falsity of its subalternate undetermined.

Or the substance of the above may be expressed in the following table:

TRUE	*A*	*E*	*I*	*O*
A		F	T	F
E	F		F	T
I	D	F		D
O	F	D	D	

F means *false;* T, *true;* and D, *doubtful.*

In this connection students need to be cautioned against thinking that the affirmation of the truth of a particular proposition necessarily implies the denial of its subalternans. In logic we usually mean *some* as the sign of a particular proposition in the sense of *some at least* rather than of *some only.* The latter is a common colloquial usage that must be kept distinct from the former.

55. Transformation of Propositions.—*Transformation of propositions is the process of stating the implied relationships between the concepts employed in them without appealing to induction to establish further data for our judgment.* In other words, the knowledge from which we start in all forms of transformation is that which is asserted in the proposition which is to be trans-

formed. Hence transformation is really nothing but a method of making more obvious and explicit, of bringing out into full relief, judgments that lurk in the background of consciousness when other judgments dealing with the same subject-matter are propounded.

There are two distinct ways of accomplishing this result. First, by establishing the relation of the subject-term to the predicate-term, there is always implied the (negative) relation of that same subject-term to the contradictory of that same predicate-term; this may be brought out in a new judgment expressing the implied relation. This is called *obversion, which consists in asserting a new judgment with the same subject-term as the original proposition. but with opposite quality, and with the contradictory of the original predicate-term as the new predicate.* Hence in obversion only the predicate-term and copula are affected. Second, a judgment establishing a relation between the two ideas represented by the two terms of a proposition implies another judgment dealing with the same subject-matter, having the same quality, and keeping within the limits of the knowledge communicated in the original proposition, but bringing out explicitly how the predicate-term is related to the subject-term in respect to distribution. This is called *conversion, which consists in interchanging the subject and predicate terms of the original proposition in the new proposition, with no change in quality.* Since the original judgment contains the knowledge from which the new proposition is derived, the *terms in the latter must never be distributed unless they were distributed in the former.* There are two types of conversion:

one, conversion *by limitation,* or *per accidens,* is so called because the subject-term of the original proposition loses its distribution in becoming the predicate-term of the new proposition. The other type is called *simple* conversion because no change in distribution occurs. There is a third species of transformation called *contraversion* (also *contraposition*), which does not introduce any new operation, *but simply combines the two already mentioned, first obverting the original proposition and then converting the proposition which results from obversion.*

The proposition from which we start in transformation is called the *obvertend, convertend,* or *contravertend,* according to the process we contemplate. The derived proposition is called the *obverse, converse,* or *contraverse,* according to the process employed to obtain it.

Obversion is applicable to all four propositions. Conversion by limitation only is applicable to A, because the predicate-term of this proposition is not distributed, and must be kept undistributed as the subject-term of the new (I) proposition. Simple conversion is applicable to E and I because there is a balance in regard to the use of the two terms in these propositions, both being distributed in E and both undistributed in I. O cannot be converted because the undistributed subject-term of the original proposition would have to become the distributed predicate-term of the derived proposition, thus violating the rule of conversion. Contraversion is applicable to all the propositions excepting I. I cannot be contraverted because the step of obversion would give O, and this cannot be submitted to the sec-

ond step of conversion for reasons already explained. Though E is contravertible, this process is not often employed upon it because its contraverse is the same as the contraverse of its subalternate O.

The following table of examples will make the recent discussion clearer:

Obversion

OBVERTEND	OBVERSE
A, *All voters are citizens,*	obverts into E, *No voters are non-citizens.*
E, *No bats are birds,*	obverts into A, *All bats are non-birds.*
I, *Some insects are winged,*	obverts into O, *Some insects are not non-winged.*
O, *Some men are not religious,*	obverts into I, *Somc men are non-religious.*

Conversion

CONVERTEND	CONVERSE
A, *All voters are citizens,*	converts into I, *Somc citizens are voters.*
E, *No bats are birds,*	converts into E, *No birds are bats.*
I, *Some insects are winged,*	converts into I, *Some winged creatures are insects.*

O is not convertible.

Contraversion

Contravertend, A, *All voters are citizens.*
(Obverse, E, *No voters are non-citizens.*)
Contraverse, E, *No non-citizens are voters.*

Contravertend, E, *No bats are birds.*
(Obverse, A, *All bats are non-birds.*)
Contraverse, I, *Some non-birds arc bats.*

Contravertend, O, *Some men are not religious.*
(Obverse, I, *Some men are non-religious.*)
Contraverse, I, *Some non-religious creatures are men.*

I is not contravertible.

The following table indicates the processes of transformation that are applicable to the several propositions:

Processes of Transformation Applicable to the Four Propositions

A	*Result*	E	*Result*
Obversion	E	Obversion	A
Conversion by limitation	I	Simple conversion	E
Contraversion	E	Contraversion	I
I		O	
Obversion	O	Obversion	I
Simple conversion	I	Contraversion	I

56. Reasons for Transforming Propositions. — The student may wonder why we should be to all this trouble to transform the various propositions. The real utility of this manipulation of the judgment will not be fully understood until the subject of deductive reasoning is reached. But a hint may be given here. In the first place, it is a powerful aid to exact thought to know and understand just how much truth, besides that directly stated, is implied in a proposition which we utter or to which we give assent. In the second place, the thinking involved in following the possible transformation of propositions and seeing the limits thereto is an excellent mental discipline, which has a value analogous to that of reducing or simplifying algebraic

expressions. In the third place, the cogency of deductive reasoning may often be more clearly demonstrated when a propositional element of the deductive argument is transformed. Let it suffice here to warn the student that the complete mastery of this topic is an indispensable condition of successful work in the analysis of argument.

REFERENCES

Creighton, *An Introductory Logic*, Ch. VII.
Hyslop, *Elements of Logic*, Chs. IX and X.
Welton, *Manual of Logic*, Vol. I, Bk. III, Ch. II, and Ch. III, § 102.
Welton, *The Logical Bases of Education*, Ch. VII.
Hibben, *Logic, Deductive and Inductive*, Chs. XII, XIII, and XIV.

REVIEW QUESTIONS

1. *How much of the denotation of the subject-term is referred to in the A proposition? of the predicate-term?*
2. *How much of the denotation of the subject and predicate terms is referred to in the E proposition?*
3. *How much of the denotation of the two terms is referred to in the I?*
4. *How much of the denotation of the subject-term is taken in the O proposition? of the predicate-term?*
5. *Why are A and E called universal? I and O particular?*
6. *How does the knowledge of the predicate-term given us in the affirmative propositions compare with that given us in the negative propositions?*
7. *What do you understand by distribution, by a distributed term, and by an undistributed term?*
8. *How many distributions are possible in the proposition?*
9. *How many distributions occur in A? in E? in I? in O?*
10. *What kind of relationship exists between A and E, and what name is given to it? Answer the same questions in respect to A and O, O and A, E and I, I and E, A and I, I and A, E and O, O and E, I and O.*

11. *Explain opposition.*

12. *Give the Square of Opposition.*

13. *Give the statement summarizing the relations of contraries; of contradictories; of subcontraries; of subalterns.*

14. *Give the table showing the effect of the truth of the several propositions upon the other propositions.*

15. *How is a particular proposition usually to be interpreted in logic? What is the usual colloquial usage of the word some?*

16. *What is transformation of propositions?*

17. *What knowledge do we start from in transformation?*

18. *How many distinct ways of transforming are there? What is the additional way and how related to these two?*

19. *Explain obversion.*

20. *What parts of the proposition are affected in obversion?*

21. *Explain conversion.*

22. *What parts of the proposition are affected in conversion?*

23. *What is the aim of conversion?*

24. *State the rule governing distribution in conversion.*

25. *Name and explain two types of conversion.*

26. *Explain contraversion, naming in their proper order the steps of which it consists.*

27. *What names are given to the original and the derived propositions in each of the processes of transformation.*

28. *To what propositions is obversion applicable? conversion by limitation? simple conversion? contraversion?*

29. *Why cannot O be converted? Why cannot I be contraverted? Why is E not usually contraverted?*

30. *Give the table showing the processes applicable to the several propositions and the results obtained.*

31. *What are the reasons for transforming propositions?*

EXERCISES ON CHAPTER VIII

1. Attach to the following propositions the letters indicating their logical character (A, E, I, or O), and place above the terms the signs (+ or −) indicating their distribution.

(*a*) *Some men are honest.*
(*b*) *No self-seeking politicians are patriotic.*
(*c*) *Some sheep are not white.*
(*d*) *All women are lovers of the beautiful.*

(*e*) *Fixed stars are suns.*
(*f*) *No mental defectives are eligible to serve in the regular army.*

2. Be prepared to explain fully, either verbally or by a figure (circles may be used to represent the concepts, being made to overlap partially, being disconnected, or one containing the other), the knowledge given respecting each of thc terms in the above propositions in relation to the other.

3. State the contrary, contradictory, and subalternate of the following:

(*a*) *All animals are breathers of oxygen.*
(*b*) *No plants are sentient.*

4. State the contradictory, subcontrary, and subalternans of the following:

(*a*) *Some insects are nocturnal.*
(*b*) *Some birds are not flying animals.*

5. Make your own example of each of the four propositions, and state the several propositions opposed to each.

6. Assuming each of the four propositions to be false, indicate in a table arranged like that on page 100 what would be the effect on the truth or falsity of the other three.

7. Obvert the following propositions:

(*a*) *All reptiles are cold-blooded vertcbrates.*
(*b*) *Some reptiles are not vertebrates with legs.*
(*c*) *No caterpillars are true worms.*
(*d*) *Some vertebrates are animals which breathe by gills.*

8. Convert as many of the propositions given in Excrcisc 7 as you think it possible to convert.

9. Contravert all the propositions given in Exercisc 7 that you deem contravertible.

10. Take each of the propositions given in Exercisc 7 and transform it by all the processes you think applicable to it, showing all steps in logical order.

11. Make your own example of each of the four propositions, and apply to it all possible processes of transformation, indicating

by the appropriate symbol all propositions formed, and showing all steps in logical order.

12. Reduce the four following propositions to their simplest logical form, and then determine the logical relation between them (six such relations exhaust the problem, viz., 1 to 2, 1 to 3, 1 to 4, 2 to 3, 2 to 4, 3 to 4; you should be able to pass by transformation from one to the other, or else to show that no such relation is possible):

(1) *All substances which are material possess gravity.*
(2) *No substances which possess gravity are immaterial.*
(3) *Some substances which are immaterial do not possess gravity.*
(4) *Some substances which do not possess gravity are immaterial.*

PART IV.—DEDUCTIVE INFERENCE

CHAPTER IX.—GENERAL NATURE OF THE INFERENTIAL PROCESSES AND THE GENERAL PRINCIPLES OF DEDUCTIVE INFERENCE

57. Nature of Inference.—What are we to understand by logical inference? This is the problem that confronts us now that the language forms, terms and propositions, are understood. *Inference from a logical stand-point is the process of passing from one judgment to a related judgment, so that the latter becomes better established as a truth by our consciousness of its connection with the former.* It is this consciousness of connection or relation upon which the validity of a logical inferring process depends. We feel and perceive the connection. There can be no doubt that this *feeling of relatedness* is an essential part of logical inference, for in it lies that emotional attitude toward the situation without which logic would have no force to win assent to the true as distinguished from the false.

Inference has two forms, *Induction* and *Deduction.* Though generally treated in logic as though they were distinct, from the view-point of modern psychology they are essentially one and the same. *Induction or inductive inference is that phase in which the judgments gain*

in generality, or, as is often said, pass from the less general (or particular) to the more general. Deduction or deductive inference is that phase in which the judgments lose in generality, or, to put it in the usual form, pass from the more general to the less general (or particular). Induction is mainly a process of *organizing* experiences into the form of accepted truth called by the general name of *knowledge,* whereas deduction is mainly a process of *applying* the organized or formulated knowledge which induction furnishes to future instances of experiences which are recognized as the same. But all application aids in completer organization; hence deduction is essential to the full and perfect organization of the knowledge which induction gathers piecemeal and puts into form. Furthermore, every instance of a deductive process of inference means that the generalization has been increased; hence deduction implies induction. Thus it appears that organization and application of knowledge proceed hand in hand—that the inductive and deductive modes of inference are only phases or aspects of one and the same mental function—that power of the mind which makes the *experienced* the indispensable agency in further *experiencing,* the *known* the essential precondition for apprehending the *unknown.* In logical inference, therefore, the mind is seen to function *inductively-deductively,* and not in either way to the complete exclusion of the other. Despite this fact, we shall have to treat the processes as though they existed independently of each other. And we shall apply ourselves to deduction first, because it shows the closest connection with the proposition and

the processes of transformation that were the subject-matter of the last chapter. This is the order demanded by pedagogical considerations, though it is not the genetic order; for all the general truths that are inferred from in deduction are the products of processes chiefly inductive.

58. Organized Experience and Deduction.— The conception of induction given above discloses its function in organizing human experience. It is necessary to have this preliminary understanding of its place in the attainment of knowledge, though the study of its processes in detail will be deferred for later treatment. By it our piecemeal mental images are united into concepts which pick out the important common phases of experience and express them in the form of widely general truths. As personal and racial experience accumulates and is handed on from moment to moment of our individual lives by memory, and from generation to generation of the racial life by spoken and written record, these truths of experience become organized into laws and principles to which is ascribed universal validity. This character of supposed universality enables the mind to extend them beyond the past, which has bequeathed them to us, to the indefinite and unborn future.

This extension of organized experience to the future can take place, however, only by an act of mental mediation. The general truth which experience has provided is applicable beyond the past only when the mind is able to identify certain elements in the instance before its attention with those found in the experiences from which the general truth has developed. It is the work

of deduction to do this identifying, or, should the identification be impossible, to declare that fact in a negative judgment. Induction has gathered up the truth embodied in the generalization, moving slowly and creepingly from fact to fact. The act of identifying a hitherto unexamined instance as essentially a repetition of former experience, or of denying this connection, is performed by deduction. From another stand-point we may view deduction as a process of interpreting a present fact by bringing forward an organized form of past experience which has characteristics in common with the new fact and hence is explanatory of it. This last statement views deduction as an elaborate form of apperception, and regards the deductive syllogism as a formal instrument for establishing the validity of the apperceptive process. A practical illustration will establish this point of view. Suppose that an analysis is made of the argument that all voluntary acts are with a view to promoting personal satisfaction, that Mr. B's gift of $200,000 to promote the education of the negroes in the South was a voluntary act, and that hence Mr. B gave the fund with a view to increase his personal satisfaction. Putting this argument into conventional syllogistic form, we have:

All voluntary acts aim to promote personal satisfaction;
Mr. B's gift for negro education is a voluntary act;
Therefore, Mr. B's gift for negro education aims to promote his personal satisfaction.

In this instance our first proposition embodies a sweeping generalization which claims the warrant of universal experience. It is a product of inductive activi-

ties reaching back into the indefinite past. Whether it be true or not is a question of the nature and teachings of a given phase of experience. To test it, resort must be had to the data of experience themselves. But admitting it for the sake of the illustration, the next act of mind, illustrated in the second proposition, is to identify (or deny the identity of) Mr. B's act of giving a gift for negro education with the class of acts known as voluntary. The moment the identification is made the conclusion follows.

It is often said that induction is going from the particular (or less general) to the (more) general, while deduction is passing from the general to the less general or particular. This is true in a sense. There is no doubt that the principle is more general in character than any one item of experience out of which it has grown. But it must be remembered that the items of experience that are the basis of a generalization lose their identity in the mental process of generalizing from them. An oak grows from an acorn, but the acorn from which a particular oak has grown cannot be placed beside it to compare with it. The mental function of generalizing or reasoning inductively is like the evolution wrought by organic growth. In the same way there is a certain error involved in saying that deduction is the process of thinking from the more to the less general. For the fact is, the more general, *i. e.*, the concept, rule, or principle, is present in the thought embodied in the conclusion no less than in the thought with which we begin. Deduction is rather a process of interpretation by which identity of nature is recognized as obtaining between an isolated item of present experience and the large sum-total of experience.

Thus it is apparent that induction prepares the way for deduction. There are no general truths until experience is fairly organized. And as soon as this organization by induction has taken place in any field of knowledge, we are supplied thereby with premises for deductive reasoning.

59. Postulates upon which Deduction is Based. —Deduction starts with certain axiomatic truths that are closely akin to those admitted in geometry. The most important of these are grouped together under the name of the *Three Primary Laws of Thought.* They are:

(1) The Law of Identity: *Whatever is, is.*

(2) The Law of Contradiction: *Nothing can both be and not be.*

(3) The Law of Excluded Middle: *Everything must either be or not be.*

The following formulas express these laws abstractly:

Identity: $A = A' = A'' = A''' \quad . \quad . \quad = A^{nth}$

or

$A = A^1 = A^2 = A^3 \quad . \quad . \quad = A^{nth}$

Contradiction: *A is not both A and not-A.*

Excluded Middle: *A is either A or not-A.*

The Law of Identity means that the essential character of anything in regard to which a judgment is made remains or abides in spite of minor changes. It does not preclude change. It merely asserts that a core of self-identity holds together the real essence or nature of the thing amid its changes. Identity is found in one and the same thing, which remains recognizable as the same amid minor changes, as when we recognize a person as the same through the changes from infancy to old age. It is also to be detected in the essential nature of the individuals constituting a class. The individuals exhibit their sameness even though it is obscured and overlaid by the differences which divide them into sub-groups and finally into individuals.

Thus in each individual example of the class horse we find a certain group of unvarying characteristics that constitute what we might call the "horseness" of horses. The first abstract formula for the law of identity is rather better adapted to exhibit the self-identity belonging to one and the same thing, passing through the changes incident to its evolution; while the second formula better expresses the identical attributes which bind together the individuals of a class.

The Law of Contradiction is the complementary of the law of identity. It places a limitation upon the latter. It virtually tells us that when change has exceeded a certain bound the changing thing ceases to be self-identical, and must thenceforth be regarded as not the same. Identity ceases at the point where one thing passes over into another. Two individuals do not belong to the same class in case the bond of likeness is too slender to overcome the difference. And when a thing can no longer show identity, it really becomes another thing. Let A change until it is no longer A, and it forthwith is not-A. It cannot be both at once. A body cannot be alive and not-alive at the same instant of time. As soon as the change takes place from one state to its contradictory state, our judgments respecting the matter in question must be entirely cast aside and replaced by others conforming to its new nature. This is but another way of recognizing that contradictory attributes cannot be applied to one and the same subject-matter in the same instant of its being. A man may be alive to-day and not-alive to-morrow, courteous in one breath and discourteous in the next, but we cannot think him

in these several contradictory conditions at any one instant of his existence.

The Law of Excluded Middle asserts the impossibility of any middle ground between contradictories. If contradictories are impossible for our thinking as applying to any subject in one instant of its being, it is equally impossible to think any third alternative lying outside of the contradictories. To have an attribute or not to have it, to be in a condition or not to be in that condition—there you have the possibilities of thought. And if thought faints at the idea of having and not having an attribute in the same phase of being, it faints no less at anything save the one or the other state. Between A and not-A judgment must choose.

Like the axioms of mathematics, these laws seem absurd at first blush. To state them seems like elaborating the obvious. Yet it is necessary for us to know what are the fundamental modes of our thinking, which logic must admit and upon which it must rely, just as the geometrician needs to know his axioms even though they are so evident that a statement of them seems superfluous. These laws or, better, axiomatic first principles of thought are not discovered by us. They are merely formulated in language and brought pointedly to our attention. Our thinking is not anywise different after we see them before us in the full dignity of print, for we have been thinking in accordance with them from the very beginning. They are just what they pretend to be on their face. There is no hidden meaning or unravelled mystery behind them. Experience does not make them any surer, for it depends upon them, not they on it.

A fourth law should be added to the foregoing. It is known as the Principle or Law of Sufficient Reason. The formulation of the law was made by the philosopher Leibnitz in the following terms:

Nothing happens without a reason why it should be so rather than otherwise.

Some of the fundamental laws of the science of physics are in reality specific applications of this law, though their "discoverers" are often not aware of the fact, mistaking them for inductive generalizations. Such is the case, for instance, with the Law of Inertia. Its universal validity is not due to experiment, but to the fact that our minds are so constituted that we cannot think a body at rest as set in motion unless a reason (cause) can be discovered, nor can we think of a moving body coming to rest unless for a reason (cause).

60. Canons of the Syllogism.—The following principles, technically known as Canons of the Syllogism, are merely a logical form of the self-evident truths already admitted in mathematics. They therefore require no explanation:

(1) *Two terms agreeing with one and the same third term agree with each other.*

(2) *Two terms, one agreeing and one not agreeing with the same third term, do not agree with each other.*

(3) *Two terms, neither of which agrees with the same third term, may or may not agree with each other.*

61. Aristotle's Dictum.—Besides these there is another self-evident truth, first stated by the Greek philosopher Aristotle, and often called the *Dictum de omni et nullo* (statement respecting all and none). It is really a fourth canon. We have already met with it in a negative form in the rule governing conver-

sion,[1] which states that it is not permissible to distribute a term in the converse unless it was distributed in the convertend. Aristotle's dictum is as follows:

Whatever is predicated affirmatively or negatively of a whole class (term distributed) may be predicated affirmatively or negatively of everything embraced within that class.

It will be observed that this is but a logical application of the axiom that what is true of the whole is true of the part.

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REVIEW QUESTIONS

1. *Define inference; induction; deduction.*
2. *What does induction accomplish in the way of organizing experience?*
3. *What must the mind do before it can extend broad general truths organized from past experience to new instances?*
4. *What does deduction do that induction alone cannot accomplish?*
5. *In what two ways may we regard deduction?*
6. *Explain the mind's inductive-deductive activity.*
7. *Memorize the Law of Identity.*
8. *Memorize the Law of Contradiction.*
9. *Memorize the Law of Excluded Middle.*

[1] See p. 101.

10. *Be prepared to express the formula for each law.*
11. *Be prepared to explain just what each law means.*
12. *How are the laws to be construed?*
13. *Memorize the Principle of Sufficient Reason.*
14. *Commit and state from memory the three canons of the syllogism.*
15. *What is another name for Aristotle's dictum?*
16. *Commit it and be prepared to state it from memory.*

CHAPTER X.—FORM AND RULES OF THE DEDUCTIVE SYLLOGISM

62. The Deductive Syllogism.—The term syllogism is derived from the Greek word *syllogismos* (συλλογισμός), which had the general meaning of reasoning, but was later technically applied by the founder of logic, Aristotle, to the deductive forms invented by him. He considered this type of reasoning, when made to conform to certain standards, to have the value of demonstration or proof. Evidently, then, the inventor of the syllogism himself regarded it not as a means of *creating* a true conclusion, as most of his followers in the Middle Ages thought it, but as a system of forms for *testing the validity* of the ordinary practical arguments that one might be drawn into when strolling with a friend or chatting with a neighbor. In accordance with this view of the deductive syllogism, we may define it as *a process of reasoning from organized knowledge to a specific instance identified with the experiences from which that knowledge was formulated, so as to demonstrate the relation conclusively.* The syllogism is therefore a standard form that all correct deductive reasoning may be made to assume and by which its validity may be tested. It *is not and does not pretend to be the usual form into which our deductive arguments are thrown.*

Deductive reasoning is called *mediate,* because the comparison of one term with the other is made through the instrumentality of a *middle* or *measuring term,* which has a determined relation to each of the terms that we are striving to compare. This being so there can be only three terms in deductive reasoning: the two compared, and the third or middle term by which they are compared. If we discover an agreement of the two with the middle term, we rely on the axiom expressed in the First Canon of the Syllogism and assert a positive relation between the terms compared. This is stated in an affirmative conclusion. If one term agrees and the other does not agree with the middle term, a relation between the terms compared is not established (according to the Second Canon of the Syllogism). This fact of no relation is expressed in a negative conclusion.

63. Form of the Syllogism.—The syllogism consists of three propositions, each containing two terms that are to appear once again. The proposition which results from and expresses the outcome of the comparison implied in the other two may usually be known by some verbal sign of inference like *therefore, hence, and so, accordingly,* etc. The first term of this proposition, which is technically called *the conclusion,* is known as the *minor term,* and the second term as the *major term,* because the reference of the latter is often broader than that of the former. The other two propositions are known as *premises*—the one containing the minor term of the conclusion being called from that fact the *minor premise,* and the other, containing the major term,

being called the *major premise.* Each premise also has the third term, called the *middle* (measuring or standard) *term,* and this does not appear in the conclusion. In a formally correct arrangement of the syllogism the *major premise is stated first, the minor premise second, and the conclusion third.* But in ordinary reasoning, even when it is fully stated, this order is not usually observed. Oftentimes the conclusion is stated first, and then the premises upon which it is grounded may be introduced by *because, since,* or similar verbal signs of the grounds of inference.

One of the typical syllogisms, in the proper form, is illustrated in the following:

MAJOR PREMISE: *All insects* (middle term) *are arthropods* (major term);

MINOR PREMISE: *All bees* (minor term) *are insects* (middle term);

CONCLUSION: *Therefore all bees* (minor term) *are arthropods* (major term).

It should be noted that the position of the major and minor terms is absolutely fixed in the conclusion, but that in the premises they may occupy either the subject or the predicate position, the place not occupied by them in the premises being taken by the middle term. This enforced variation in the position of the middle term in the premises gives rise to the several changes in form technically known as *figures.* These will be considered a little later.

64. RULES OF THE SYLLOGISM.—The following rules, obvious enough after a little thought, condition the

structure and limit the nature of the syllogism. An explanation of them will follow their statement:

I. Rules of Form:

1. *The syllogism has three terms only, once repeated.*
2. *The syllogism has three propositions only.*

II. Rules Governing the Premises:

1. *No conclusion can be inferred from negative premises.*
2. *No conclusion can be inferred from particular premises.*

III. Rules Governing the Conclusion:

1. *If a premise be negative, the conclusion is negative; and a premise must be negative to prove a negative conclusion.*
2. *If a premise be particular, the conclusion is particular.*

IV. Rules Governing Distribution:

1. *The middle term must be distributed at least once.*
2. *No term may be distributed in the conclusion unless distributed in its premise.*

It follows as a corollary from Rule 1, under I, that none of the terms may be ambiguous; for any term which is ambiguous, since it refers to different things in its two uses, really makes a syllogism of four instead of three terms.

65. The Rules Explained.—It will be well to make the reasons for these rules clear and explicit, though these reasons are already implied in what has been said regarding the structure and canons of the syllogism.

I. (1) The syllogism must have as many as three terms, because its very purpose is to bring two terms into comparison by means of their definite relations to a third (middle) term. It can have no more than three terms because the common relation of the two to a third could not be established were there more than the three. To make this clear let us suppose four terms, designated as A, B, C, and D. If in one premise A were related to B, and in the other C to D, we should be without any common standard (middle term) through which to reach a conclusion. If we really secured a middle term we should have two syllogisms—first, one where A is related to B (middle) in one premise, and C to B in another, leading to a conclusion regarding the relation of A and C; and second, another where C is related to A (the conclusion of the first syllogism) in one premise, and D to A in another, leading to a conclusion regarding the relation between D and C.

I. (2) Since the propositions are the means of relating two terms, and since only three terms are compared, it is obvious that there can be only three propositions, one comparing one term with the middle term, another comparing the other term with the middle term, and a third stating the relation between the two terms which have been compared with the middle term.

II. (1) The Third Canon of the Syllogism shows that no knowledge as to the relation between two terms can be obtained when there is no agreement. Hence, in the absence of any agreement with a common term, there can be no conclusion.

II. 2. Two particulars fail to give a conclusion be-

cause of difficulty in the distribution of terms. The only possible combinations of premises are I I, I O, O I. O O is impossible because it violates the rule against negative premises. Of the other combinations, I I would be impossible, for since the I distributes neither term there is no opportunity for distributing the middle term (*cf.* Rule 1, IV). I O and O I would both lead to a particular negative (O) conclusion (Rules 1 and 2, III). The O conclusion distributes one term (the predicate), and hence two distributions must be provided in the premises, one for the middle and one for the major. But only one place (the predicate of the O) is provided for distribution in any combination of I and O. Hence there would be a violation of either Rule 1 or 2, IV.

III. (1) This rule is based upon the canon that where one term agrees and the other disagrees with a third (middle) term, they disagree with each other (expressed in a negative conclusion). The requirement of a negative premise to prove a negative conclusion is explained by the fact that a negative conclusion expresses a case of disagreement between terms, and it is manifest that, since in general the conclusion depends upon the nature of the premises, no such conclusion can be drawn unless a disagreement has been expressed in one of the premises.

III. (2) This rule involves the principle of distribution of terms already met with in conversion (see p. 101), and stated in its application to the syllogism in Rules 1 and 2, IV. The only possible combinations involving a particular premise are A and O or I, and E and I. O and I, O and O, and I and I are precluded by Rule 2, II;

E and O by Rule 1, II. We shall resort to indirect proof to show that the possible combinations of premises must give a particular conclusion. Suppose the conclusion universal, then it would be either A or E. If A, there would have to be one distribution in the premises besides that providing for the middle term, for the A conclusion distributes its minor term. If E, there would have to be two distributions in the premises besides that providing for the middle term, for the E conclusion distributes both its minor and major terms. In all, then, the premises would have to make two distributions were the conclusion A and three were it E. Now the combination of A and I as premises provides only one distribution (the subject of the A), and hence would be impossible. The other combination, A and O, would provide two distributions, but the only universal conclusion permissible would be E (A would violate Rule 1, III), which would require *three* distributions. Hence A in combination with any particular as premises could not give a universal conclusion. The only combination not yet tested is E and I. The conclusion could not be A, for this violates Rule 1, III. If it were E, which distributes both terms, the premises would have to provide three distributions (two for the major and minor, and one for the middle). But I and E in combination provide only two distributions. Hence we could not have a universal conclusion from I and E.

IV. (1) The reason for requiring the distribution of the middle term once at least is apparent, when one considers that if not distributed in either premise, there is

no means of instituting an intelligent comparison of the major and minor terms. The major term might allude to things lying in one part of the denotation signified by the middle term, while the minor term might allude to things lying in an entirely different part of its denotation. This condition of things would give no ground for bringing the minor and major terms into positive or negative relation in the conclusion. Violation of this rule occasions the fallacy of Undistributed Middle. The following fallacious syllogism illustrates how absurd would be a conclusion reached from premises providing no distribution of the middle term:

All eagles are birds (undistributed middle);
All finches are birds (undistributed middle);
Therefore all finches are eagles.

IV. (2) The rule prohibiting the distribution of terms in the conclusion unless they were distributed in the premises is merely an extension of the principle already explained (p. 101) as conditioning conversion. Just as in conversion the convertend is to be regarded as an empirically derived general truth, stating all the knowledge regarding the relation of the terms that experience warrants; so in the syllogism the premises are the ground, guaranteed by experience, of the judgment expressing the conclusion. This once admitted, it is immediately apparent that the terms as used in the conclusion must keep within the denotation signified by the terms as employed in the premises. Violation of this rule involving the minor term issues in the fallacy of *illicit process of the minor term* (*illicit minor*), involving

the major term, in the fallacy of *illicit process of the major term* (*illicit major*).

It should here be noted that the rules under headings II and III condition the *moods,* and those under IV the *figures* of the syllogism. The meaning of these terms will be made clear in the ensuing explanation.

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Sigwart, *Logic,* Vol. I, Pt. II, Ch. III, § 55.

REVIEW QUESTIONS

1. *What is the derivation of the word syllogism?*
2. *How did Aristotle use it?*
3. *How may the deductive syllogism be defined?*
4. *What relation has the syllogism to the common form of reasoning?*
5. *Why is deductive reasoning called mediate?*
6. *How many terms can there be to a true reasoning process?*
7. *What is the conclusion when both premises state an agreement?*
8. *What is the conclusion when one premise states a disagreement?*
9. *Under what circumstances can no conclusion be reached?*
10. *How many propositions in the syllogism?*
11. *How many times is each term used in the syllogism?*
12. *What term appears only in the premises?*
13. *Where else would you look for the subject-term of the conclusion? for its predicate-term?*
14. *What is the major term? the minor term? the middle term?*
15. *Where is the minor term fixed in its position? where free to change its position? Answer the same questions about the major term.*

16. *Name some common verbal signs of a conclusion? of the grounds of inference?*

17. *What is the formal or standard order of the syllogism? Does ordinary reasoning follow this order?*

18. *Upon what does the position of the middle term depend? To what does this position give rise?*

19. *Memorize and be prepared to state all the rules of the syllogism (just as stated in the text).*

20. *Be prepared to show the reason for or to prove each of the rules.*

21. *What occasions undistributed middle? illicit major* and *minor?*

22. *Which rules condition the moods? which the figures?*

EXERCISES ON CHAPTER X

1. Select and name the various propositions and terms of the syllogism in the following informally stated arguments. Then reconstruct them so as to make formally correct syllogisms:

(1.) *All leeches must be true worms; for all annelids are worms, and leeches are annelids.*

(2.) *Caterpillars have true legs; worms do not; and so caterpillars are not worms.*

(3.) *Amphibians are not reptiles, since they breathe by gills in the larval stage, and reptiles do not.*

2. Using *dogs* as the middle term, construct a formally correct syllogism proving that *all hounds are flesh-eaters.* Name each term and proposition as used in the syllogism.

3. With *insects are invertebrates* as a major premise, supply a minor premise to prove that *butterflies are invertebrates.*

4. With *some political bosses do not seek to appropriate property without labor* as a minor premise, supply a major premise that would bring the conclusion that *some political bosses are not dishonest.*

5. Construct a syllogism with *eagles* as the minor term, *birds of prey* as the middle term, and *capable of sustained flight* as the major term, leading to an A conclusion.

6. Construct a syllogism with *fish* as the major term, *mammals* as the middle term, and *whales* as the minor term, giving an E conclusion.

CHAPTER XI.—MOODS, FIGURES, AND REDUCTION

66. Permutation of Propositions in the Syllogism.—There are four kinds of propositions (A, E, I, and O) and three places for them in the syllogism. Ignoring for the moment the conflict of certain combinations with the rules of the syllogism, and assuming that they may combine in any way and order, these four propositions in the three places which the syllogism offers to them would present sixty-four permutations or changes of combination before there would be a repetition of form. That is, the possible permutations without repetition would be mathematically represented by four raised to the third power (4^3). To these permutations is given the technical name of *moods*. By mood, then, is meant *the kind of propositions entering into the construction of the syllogism and the order in which they are used.* It is purely a mechanical procedure to write out all possible permutations. To do so one has simply to begin with the A proposition as a major premise, A as a minor premise, and A as a conclusion (A A A). Giving heed to the order of the letters A E I O in the alphabet, change the conclusion four times (until all possible conclusions have been written with A A as premises). Next change the minor premise to E, writing the four conclusions with A E as premises. Continue changing the minor premise, with A still the major

premise, until all possible combinations with A as a major premise have been exhausted (there will be sixteen cases). Then change the major premise to E, and proceed as before directed (simply copy the series of minor premise-conclusion combinations already written with A as the major premise). Then take I as the major premise, and so on. An examination of a complete table of permutations discloses the fact that many of the moods violate rules of the syllogism (those under headings II and III), and hence cannot be used. Our next task is consequently to determine which of these sixty-four permutations are *valid moods.* The simplest way to do this is first to examine the premises to see which of them must be rejected because they break rules under II (p. 123).

67. Valid Premises.—There are sixteen possible combinations of premises resulting from the permutations of A, E, I, and O, to wit:

A A	E A	I A	O A
A E	E E	I E	O E
A I	E I	I I	O I
A O	E O	I O	O O

E E, E O, O E, and O O conflict with Rule 1, II.
I I, I O, O I, and O O conflict with Rule 2, II.

Hence the remaining valid premises are as follows:

A A	E A	I A	O A
A E	E I	I E	
A I			
A O			

It is interesting to note that of the valid premises there are four cases with a universal affirmative (A) major,

one with a particular negative (O) major, and two each with a universal negative (E) and a particular affirmative (I) major. This implies the greatest value in knowledge that refers to the full denotation of a term, and is assertive rather than denying in character.

68. Valid Conclusions from the Valid Premises. —The task to which we must next address ourselves is to find the conclusions from these nine valid premises to which the rules under III commit us. This can best be presented in a table showing the valid premises, the conclusions which they permit, and the rules conditioning just those conclusions.

PREMISES				CONCLUSION	RULE
	A A			A or I	1, III.
	A E	E A		E or O	1, III.
	A I	I A		I	1 and 2, III.
A O	O A	E I	(I E)[1]	O	1 and 2, III.

69. Table of Valid Moods.—The following eleven moods are all that can be used in valid argument. That is to say, any possible deductive reasoning must have propositions that are capable of being made to conform to one of the following arrangements:

A A A	E A E	I A I	O A O
(A A I)	(E A O)		
A E E	E I O		
(A E O)			
A I I			
A O O			

[1] I E O does not violate any of the rules cited in the foregoing table, but it is invalid because of illicit major. For the conclusion, O, requires the distribution of the major (predicate) term. But the major premise is I, which distributes neither term; hence the major term cannot be distributed in its premise.

The moods in parenthesis are called *weak moods* when employed in a figure that would allow a universal conclusion to be inferred from the same premises. Thus I from the premises A A is really merely the subalternate of the conclusion A that these same premises allow us to draw. This leaves eight really distinct types of valid moods (omitting those in parenthesis). It will be found, however, that the moods A A I and E A O are the only ones with those premises that can be used in the Third and Fourth Figures.

It is interesting to notice that A as a major premise enters into combination with all four forms of proposition as a minor premise, and yields all four propositions as conclusion. E as a major combines only with A and I as minors, and gives only negative conclusions. I as major combines only with A as minor, giving only I as conclusion. O as a major combines only with A as a minor, giving only O as a conclusion. These facts seem to indicate what our ordinary experience bears out, that a judgment respecting the full denotation of the subject-term, and affirming something of that term (universal affirmative), furnishes the best possible apperceptive basis into relation with which as an interpreting medium we may bring the present situation needing interpretation. The minor which presents the present situation needing explanation may under these favorable circumstances be any form of judgment. And any of the four judgments may be reached as conclusions. This would indicate that knowledge should if possible be organized into a form permitting expression as A judgments.

70. Figures.—Up to this point little or nothing has been said of the position of the terms in the premises. All that we know is that the major term is found in the major premise, the minor term in the minor premise, and the middle term in each premise. Each premise offers two positions to the middle term—it may be either subject or predicate. And likewise there are two positions open in each premise to the other terms. Since the middle term occurs in both premises, there are four permutations in its position. By figure is meant *the position of the middle term in the two premises.* Evidently there are four possible figures—one where the

middle term is subject of the major premise and predicate of the minor premise; another where it is predicate of both premises; another where it is subject of both premises; and a last where it is predicate of the major premise and subject of the minor premise. To these permutations the names *First, Second, Third,* and *Fourth Figures* are given respectively.

The following symbolical devices may be used to represent the several figures and will be found of assistance to the memory:

	FIRST		SECOND		THIRD		FOURTH	
	S.	P.	S.	P.	S.	P.	S.	P.
MAJOR PREMISE	·	*	*	·	·	*	*	·
MINOR PREMISE	o	·	o	·	·	o	·	o

The * stands for the major term, the ° for the minor term, and the · for the middle term. *S* stands for subject-term and *P* for predicate-term. To make the meaning perfectly clear, we may take a pair of premises in the First Figure and apply the device for that figure to them, placing the symbols representing the terms just above them.

MAJOR PREMISE. *All* $\overset{\cdot}{\textit{men}}$ (middle term) *are* $\overset{*}{\textit{mortal}}$ (major term);

MINOR PREMISE. *All* $\overset{\circ}{\textit{Greeks}}$ (minor term) *are* $\overset{\cdot}{\textit{men}}$ (middle term).

71. MOODS VALID IN THE SEVERAL FIGURES.—When the figures are considered in connection with the ways the several propositions distribute their terms (table p. 95), and also in view of the requirements imposed by Rules of the Syllogism 1 and 2, IV, it will be

evident that not every valid mood is adapted to all the four figures. It now remains to see what moods may be validly used in each of the four figures. The following table exhibits all the valid moods under each of the four figures. The weak moods are placed in parenthesis. In some figures the weak conclusion is the only one that can be deduced, though a different order of premises would allow a universal conclusion in another figure. In other instances the conclusion may be universal, and hence we sacrifice the knowledge that the premises will give by accepting the weak conclusion in place of the universal one.

FIRST FIGURE	SECOND FIGURE	THIRD FIGURE	FOURTH FIGURE
A A A	A A A[1]	A A A[2]	A A A[2]
(A A I)	(A A I)[1]	(A A I)	(A A I)
A E E[2]	A E E	A E E[2]	A E E
(A E O)[2]	(A E O)	(A E O)[2]	(A E O)
A I I	A I I[1]	A I I	A I I[1]
A O O[2]	A O O	A O O[2]	A O O[1]
E A E	E A E	E A E[2]	E A E[2]
(E A O)	(E A O)	(E A O)	(E A O)
E I O	E I O	E I O	E I O
I A I[1]	I A I[1]	I A I	I A I
O A O[1]	O A O[2]	O A O	O A O[2]

The index numeral 1 indicates that the moods after which it is placed are in conflict, in the given figure, with Rule of the Syllogism 1, IV (see p. 123); the numeral 2 indicates that those referred to by it are in conflict with the Rule of the Syllogism 2, IV. Those moods marked 1 would if used give rise to undistributed middle; those marked 2 to illicit minor or major. For example, A A A in the second figure would place its middle term in the predicate's place in both premises; but as the propositions forming the premises are both A's, the middle term would not be distributed. A E E in the first figure would be illicit major; for the conclusion, being E, distributes both its terms, hence the predi-

cate or major term. But this term occurs in an A major premise, which distributes only its subject, and in the first figure the subject of the major premise is occupied by the middle term. Therefore there is no opportunity to distribute the major term in the major premise in order to admit of its distribution in the E conclusion. Other cases can be tested after this fashion by the student and the ground of their rejection proved to his satisfaction.

The following is a list of the moods valid in the several figures after those that are not adapted to particular figures have been eliminated. The spacing indicates rejections.

FIRST FIGURE	SECOND FIGURE	THIRD FIGURE	FOURTH FIGURE
A A A			
	A E E		A E E
A I I		A I I	
	A O O		
E A E	E A E		
E I O	E I O	E I O	E I O
		I A I	I A I
		O A O	
		A A I[1]	
	(Weak moods)		
(A A I)			(A A I)[2]
	(A E O)		(A E O)
(E A O)	(E A O)	(E A O)[2]	(E A O)[2]

It is worthy of note that the first figure is the only one giving all the four propositions as conclusions; that the second gives only negative conclusions; that the third gives only particular conclusions; and that the fourth does not give a universal affirmative conclusion. The fourth was regarded by Aristotle, who

[1] The conclusion, I, in this mood can just as well be inferred from the premises A I as from A A.

[2] These premises give universal conclusions when arranged in the first figure.

established logic, as awkward, and was not recognized until the time of Galen (died about 200 A. D.), whence it is called the Galenian figure.

72. Special Canons of the Figures.—A critical inspection of the several figures will reveal the fact that each is subject to a special canon, or group of canons, which condition to some degree its propositions. These canons must now be considered.

First Figure.—A negative minor premise would result in a negative conclusion (Rule of the Syllogism 1, III). A negative conclusion distributes its predicate (major) term, and hence the major term must also be distributed in the major premise (Rule of the Syllogism 2, IV). But as the major term occurs as the predicate in the first figure, and as only negative propositions (E and O) distribute their predicate-terms, we should have to have a negative major premise. But our minor premise is by supposition also negative. Hence we should be trying to deduce a conclusion from two negative premises, in violation of Rule of the Syllogism 1, II. Therefore in this figure the minor premise cannot be negative, hence must be affirmative.

The major premise cannot be particular in the first figure; for, since the minor premise is affirmative, its predicate, where the middle term occurs in this figure, is undistributed (neither A nor I distribute the predicate-term). Hence the distribution of the middle term must be provided for in the major premise (Rule of the Syllogism 1, IV), where it occurs as subject in the first figure. But as particulars (I and O) do not distribute their subject, the major premise must be universal to do so.

Special Canon of the First Figure: *In the First Figure the minor premise must be affirmative and the major premise universal.*

Second Figure.—In this figure the middle term occurs in the predicate of both premises. Hence in order to provide for its distribution one premise must be negative (for neither affirmative proposition distributes its predicate). Since a premise must be negative, the conclusion will be negative (Rule of the Syllogism 1, III). Hence the major term must be distributed in the major premise (the negative conclusion will distribute its major term, and there must be provision for this distribution in the major premise; otherwise there is a violation of Rule of the Syllogism 2, IV). The major term occurs as the subject of the major premise in this figure. Hence to distribute it the major premise must be universal (neither particular distributes its subject).

Special Canon of the Second Figure: *In the Second Figure one premise must be negative, the conclusion therefore negative, and the major premise universal.*

Third Figure.—An inspection of this figure shows that the minor premise cannot be negative for exactly the same reason as explained in the case of the first figure (see above). Since the minor premise must be affirmative its predicate will be undistributed (neither affirmative distributes the predicate term). In this figure the predicate of the minor premise is the situation of the minor term, which is undistributed in its premise. Therefore it may not be distributed in the conclusion (Rule of the Syllogism 2, IV). But both universals

distribute the subject-term; therefore the conclusion (as subject of which the minor term occurs) must be particular.

SPECIAL CANON OF THE THIRD FIGURE: *In the Third Figure the minor premise must be affirmative and the conclusion particular.*

Fourth Figure.—By inspection similar to that demonstrated above for the foregoing figures, each student may prove for himself the following canon for the fourth figure.

SPECIAL CANON OF THE FOURTH FIGURE: *In the Fourth Figure, if a premise be negative, the major premise must be universal; if the major premise be affirmative, the minor premise must be universal; and if the minor premise be affirmative, the conclusion must be particular.*

73. REDUCTION.—It is a tradition of logic, traceable to its founder Aristotle (384–322 B. C.), that deductive demonstration is most easily followed in the first figure, because there the *Dictum de omni et nullo* (see p. 118) is most applicable, especially in mood A A A. While modern logic has exploded this idea, it is still customary to retain a place for an explanation of the method of transforming an argument from the second, third, and fourth figures to the first. Aside from historical interest, exercises of this character serve to familiarize the student with the structure and limitations of the syllogism, and also furnish excellent drill in abstract thinking. This process has received the name of reduction. It is of interest to notice that it is possible to "reduce" from any figure to any other, though custom limits the process to the forms mentioned above.

Upon consulting the diagram of figures on p. 134, it is seen that the second figure becomes the first by putting the middle term of the major premise in the subject's place, the third becomes the first by putting the middle term of the minor premise in the predicate's place, and the fourth becomes the first by putting the middle term of the major and minor premises in the subject's and predicate's places respectively. It must be understood that the limitations imposed upon one are not to pass beyond the logical value of the premises as originally granted. It will be recalled that the method of changing the positions of terms without passing the limits of the original judgment is some form of transformation (*cf.* pp. 100–104). The process most available for the purpose in hand is conversion. But two facts must be kept in mind: one is that O cannot be converted, the other that A converts by limitation into I. Hence we are checkmated if the situation requires the transformation of an O proposition, which happens in the two moods having O premises (A O O, 2d fig. and O A O, 3d fig.); and here resort must be had to a special process known as indirect proof, to be explained later on. We are also confronted with difficulty if an A requires conversion when used with a particular premise or when followed by a universal conclusion; for in the former event the conversion of A to I would cause two particular premises and no conclusion, and in the latter one particular premise, and hence a particular conclusion, whereas a universal one is desired.

All problems in reduction may, then, be classified under three heads: first, where only conversion is re-

quired; here are included all instances where the premise (both premises in the fourth) to be converted is E, I, or A, with the other premise universal and the conclusion particular; second, where the premises must be transposed in order to bring an E or an I into the place requiring conversion, and the conclusion also converted to correspond with the new positions of terms in the new order of premises; this includes all cases excepting those noted under the first and third heads in the following table; third, A O O and O A O, requiring indirect proof. The cases may be presented in a table:

CONVERSION	TRANSPOSITION AND CONVERSION	INDIRECT PROOF
2d fig.	*2d fig.*	*2d fig.*
E A E	A E E	(A E O)
(E A O)		A O O
E I O		
3d fig.	*3d fig.*	*3d fig.*
A A I	I A I	O A O
A I I		
(E A O)		
E I O		
4th fig.	*4th fig.*	*4th fig.*
(E A O)	(A A I)	(A E O)
E I O	A E E	
	I A I	

The following points must be observed: The moods in parenthesis are "weak"; transposition of premises in the fourth figure effects arrangement in the first figure without any conversion except that of the conclusion to correspond to the new major and minor terms; the A A premises of mood A A I of the fourth figure when transposed yield an A conclusion; A E O (2d and 4th figures) presents a technical difficulty requiring indirect proof, viz., if we convert the A there results I E O, which was rejected

on account of illicit major (*cf.* p. 132, note), while if we transpose the premises there is the need of converting the conclusion, which is O, and so inconvertible; hence we have to use indirect proof or else prove the truth of the O as the subalternate of the E conclusion given by mood A E E in the second and fourth figures. Usually the mood will be changed when transposition of premises is necessary, thus A E E, second figure, becomes E A E in the first.

The following examples illustrate the forms of reduction:

E A O (3d) to E I O (1st).

No Republicans are advocates of free-trade;
All Republicans are interested in tariff reform;
Therefore some who are interested in tariff reform are not advocates of free-trade.

Converting the minor premise A to I puts the syllogism in the first figure, thus, *Some who are interested in tariff reform are Republicans.*

A E E (2d) to E A E (1st)

All fish are cold-blooded;
No whales are cold-blooded;
Therefore no whales are fish.

Transposing the premises and converting both E's we have:

No cold-blooded animals are whales;
All fish are cold-blooded;
Therefore no fish are whales.

A O O (2d) by indirect proof

All voters are citizens;
Some residents of New York are not citizens;
Therefore some residents of New York are not voters.

If this conclusion is challenged, its contradictory, *All residents of New York are voters*, is true (*cf.* Summary of Relations, No. 2, p. 100). But if the contradictory, A, *is* true, it should give the other premise as a conclusion when used as a premise with either of the original premises. Try it with either premise that puts the syllogism in the first figure (to use any other figure would be proving in a circle, since the first conclusion was doubted because the argument was *not* in the first figure). The trial syllogism produces a conclusion which is the *contradictory* of one of the original premises; and since the truth of these has already been granted, the contradictory of the first conclusion cannot be the true conclusion from the original premises, whereas the first conclusion was the true inference. This is evident in the following:

All voters are citizens;
All residents of New York are voters;
Therefore all residents of New York are citizens.

But the truth of the premise, *Some residents of New York are not citizens*, was granted in the beginning; hence this last conclusion, *All residents of New York are citizens*, which contradicts it, cannot be true. But it follows in a valid syllogism which can be attacked only on the ground that a premise is false. The false premise is the substituted conclusion, whose contradictory (the original conclusion) must be true (*cf.* Summary of Relations, No. 2, p. 100), *q. e. d.*

REFERENCES

Creighton, *An Introductory Logic*, Ch. IX.
Welton, *Manual of Logic*, Vol. I, Bk. IV, Chs. III and IV.
Hyslop, *Elements of Logic*, Chs. XII and XIII.
Hibben, *Logic, Deductive and Inductive*, Pt. I, Ch. XVI.
Minto, *Logic, Inductive and Deductive*, Bk. I, Pt. IV, Chs. II and III.
Aikins, *The Principles of Logic*, Chs. XII–XVI.

REVIEW QUESTIONS

1. *How many are the possible permutations of the four propositions in combining to form the syllogism?*
2. *Define mood.*
3. *What are the two steps in determining the valid moods?*
4. *How many possible combinations of premises are there?*
5. *Name the valid premises.*
6. *What proposition as major premise allows of the most combinations? what the least?*
7. *What conclusions must follow from each valid pair of premises?*
8. *Explain the rejection of I E O.*
9. *Develop in alphabetical order the strong valid moods; the weak valid moods.*
10. *Explain the meaning of strong mood.*
11. *Compare the conclusions resulting from premises of which A is the major; E the major; I the major; O the major.*
12. *What does this comparison show as to the value of the A judgment as a form of knowledge?*
13. *What positions are open to the middle term in each premise?*
14. *How many permutations in the combination of middle term in the premises are there?*
15. *Define figure.*
16. *How many figures are there, and where is the middle term in each?*
17. *Locate the major and minor terms in each of the figures.*
18. *How are the terms distributed in each of the four propositions?*
19. *Develop the strong moods valid in each figure; the weak moods.*

20. *Which figure gives all four propositions as conclusions? Which only negative conclusions? Which only particular conclusions? What conclusion does the fourth figure fail to give? Assign reasons in each case.*

21. *How did Aristotle regard the fourth figure? By whom was it restored to favor?*

22. *State each of the special canons of the figures.*

23. *What is reduction?*

24. *What figures do we reduce from? What do we reduce to?*

25. *What principle must be observed in reduction? Where has it been met with before?*

26. *How do the first and second, first and third, and first and fourth figures differ?*

27. *Why cannot we reduce A O O and O A O?*

28. *What did Aristotle think of the first figure?*

29. *Why is reduction still considered a part of the study of the deductive syllogism?*

30. *What is the real advantage to be derived from reduction?*

EXERCISES ON CHAPTER XI

Note.—In making illustrative syllogisms having negative propositions the student is cautioned to avoid two common errors: first, the error of confusing the forms of the E and the O propositions (here review carefully what was said on this point on p. 77, fine print); and, second, the absurdity of negating a relation between a subject-term and a predicate that would not be thought in connection with it, as, for example, in the proposition, *No dogs are hens.* All negating means a drawing of distinctions where the mind might without care confuse classes. So that there is a sense in which negation implies a higher affirmation that reconciles it. Thus we often find it useful to negate as between species of the same genus, but not as between sub-classes of a genus that is remotely distant above them. To say, *No elephants are watermelons*, is true enough, forsooth, but it sounds like the rankest folly; for the likeness (namely, that they are both living organisms) is so remote, the genus that would embrace the two classes so remotely above them, that confusion could not occur. On the other hand, it is illuminating to assert, *No spiders are insects*, because the classes may both be regarded as (logical) species of the genus arthropods, and this

fact makes them both the possessors of striking conferentia which might easily obscure the differentia that hold them apart as distinct kinds.

1. Write out the sixty-four permutations alphabetically.
2. Write out the sixteen permutations of propositions in the premises.
3. Reject all the permutations in the premises that are invalid, citing the rule governing each rejection.
4. Cite the rules in each instance by which we are limited to the conclusions which follow from the valid premises.
5. Refer to the table on p. 95 showing distribution of terms, use + or − over the term to indicate its character in regard to distribution, and cite the proper rule of the syllogism (1 or 2, IV), in order to explain whether each instance of rejection of a mood from a figure is a case of undistributed middle, illicit minor, or illicit major.
6. Make a false syllogism in illustration of each of the moods rejected from the several figures.
7. Make a syllogism in illustration of each of the strong moods valid in the several figures.
8. Prepare to prove the special canon of the fourth figure.
9. Reduce all the valid syllogisms in the second, third and fourth figures, that were made in answer to Exercise 7, to the first figure, showing clearly all transformations and transpositions.
10. Make an A O O (2d figure) and an O A O (3d figure) and prove indirectly.
11. Make examples of the syllogisms in weak moods and reduce to the first figure, getting a universal conclusion when possible.
12. Make a syllogism in E I O, any figure, and exhibit the conversions necessary to accommodate it to each of the other figures.
13. Prove some one mood (in addition to A O O and O A O) from each figure by indirect proof.
14. Make a syllogism in A I I, first figure, and accommodate it to the third (A I I) and fourth (I A I) figures.
15. Make a syllogism in E A E, first figure, and accommodate it to the second and fourth figures, naming the mood obtained and showing all steps.

CHAPTER XII.—HYPOTHETICAL AND DISJUNCTIVE SYLLOGISMS AND THE DILEMMA

74. Conditional Syllogisms.—Up to this point only categorical propositions have been regarded as going to form the syllogism. It has already been explained [1] that judging may be conditional and give rise to the conditional form of proposition. This proposition may enter into the composition of the syllogism. The conditional judgment may be either hypothetical or disjunctive. The hypothetical judgment states a supposition, and is usually introduced by the words *if*, *in case*, *provided that*, etc. The disjunctive judgment states alternatives, and is most commonly expressed by the disjunctive words *either . . . or*.

75. Hypothetical Form of Conditional Syllogism.—In a conditional proposition of the hypothetical form the clause introduced by *if* or its synonyms is called the *antecedent*, while the clause stating the result on the condition instanced is known as the *consequent*. In the syllogism the conditional proposition occurs only as major premise. The minor premise is categorical. It either affirms the antecedent or denies the consequent. In the former case it leads to an affirmative conclusion, which asserts the consequent to be true; in the latter case, to a negative conclusion, which denies the truth

[1] See p. 71 *f*.

of the antecedent. The one is called a *constructive hypothetical syllogism,* and the argument expressed by such a syllogism is said to be of the *modus ponens,* or mood which posits or affirms; the other is called a *destructive hypothetical syllogism,* and the argument expressed is spoken of as *modus tollens,* or mood that removes the consequent.

EXAMPLES:

CONSTRUCTIVE HYPOTHETICAL SYLLOGISM

(*Modus ponens*)

If these creatures have six legs, they are insects;
But they have six legs;
Therefore they are insects.

DESTRUCTIVE HYPOTHETICAL SYLLOGISM

(*Modus tollens*)

If these creatures have six legs, they are insects;
But they are not insects;
Therefore they have not six legs.

76. REDUCTION OF THE HYPOTHETICAL TO THE CATEGORICAL SYLLOGISM.—An examination of the hypothetical proposition shows that the same meaning can usually be expressed in a categorical form. For example, the proposition, *If these creatures have six legs, they are insects,* is equivalent in logical value to the A proposition, *Six-legged creatures are insects.* Using this as a major premise, we have as a substitute for the constructive syllogism:

Six-legged creatures are insects;
These are six-legged creatures;
Therefore these are insects.

And for the destructive syllogism the substitute:

Six-legged creatures are insects;
These are not insects;
Therefore these are not six-legged.

The first is in this new form A A A (1st fig.), and the second A E E (2d fig.).

Oftentimes the change from a hypothetical to a categorical form demands a somewhat awkward use of language. Thus in the sentence, *If Plato is right, communism is the best form of society,* adaptation to the categorical form compels us to use some such awkward expression as this, *The case of Plato's being right is the case of communism's being the proper form of society.* But in all instances some mode of expression can be invented to put the thought in categorical form.

77. Fallacies of the Hypothetical Syllogism.— A moment's examination of the hypothetical proposition, with a view to a clear determination of its meaning, will show us the fallacies that are liable to arise when it is used in the syllogism, and will also give us the rule governing its proper use. In the sentence, *If a man is born in France, he is a European,* the condition named, *birth on French soil,* is not the only condition of his being a European. One could also say with equal truth, *If a man is born in England, if a man is born in Germany, etc., he is a European.* In other words, while all cases of birth in France are cases of being Europeans, the converse is not true. This amounts to saying that the antecedent, *if a man is born in France,* is distributed, while the consequent, *he is a European,* is undistributed, as appears at once when we state the meaning in simple categorical form (A), *All Frenchmen are Europeans.* Suppose, now, that with the above-

stated hypothetical proposition as a major premise we should take as minor premise the proposition, *But this man was not born in France,* and reason to the conclusion, *Therefore he is not a European,* it is evident that we should be guilty of a fallacy that is readily seen to be illicit major when the syllogism is stated in categorical form, as follows:

All instances $\overset{+}{\textit{of}}$ *birth in France are instances* $\overset{-}{\textit{of}}$ *Europeans;*
(*All* $\overset{+}{\textit{Frenchmen}}$ *are* $\overset{-}{\textit{Europeans}}$)
$\overset{+}{\textit{This}}$ *is not an instance* $\overset{+}{\textit{of}}$ *birth in France;*
(*This man* $\overset{+}{\textit{is}}$ *not a* $\overset{+}{\textit{Frenchman}}$)
Therefore $\overset{+}{\textit{this}}$ *is not an instance* $\overset{+}{\textit{of}}$ *a European;*
(*Therefore this* $\overset{+}{\textit{man}}$ *is not a* $\overset{+}{\textit{European}}$).

This is the mood A E E and the first figure, whereas we know that this mood in the first figure gives illicit major.

On the other hand, suppose that with the same major premise we use the minor premise, *But this man is a European,* and reason to the conclusion, *Therefore he was born in France,* we should have what the categorical form of syllogism shows to be undistributed middle, as the following reconstruction in categorical form shows:

All instances $\overset{+}{\textit{of}}$ *birth in France are instances* $\overset{-}{\textit{of}}$ *Europeans;*
(*All* $\overset{+}{\textit{Frenchmen}}$ *are* $\overset{-}{\textit{Europeans}}$)
$\overset{+}{\textit{This}}$ *man is an instance* $\overset{-}{\textit{of}}$ *a European;*
($\overset{+}{\textit{This}}$ *man is a* $\overset{-}{\textit{European}}$)
Therefore $\overset{+}{\textit{this}}$ *man is an instance* $\overset{-}{\textit{of}}$ *birth in France;*
(*Therefore* $\overset{+}{\textit{this}}$ *man is a* $\overset{-}{\textit{Frenchman}}$).

This is the mood A A A in the second figure, and the middle term, *instance of Europeans*, is undistributed in both premises. Of course A A A is not permissible in this figure.

78. RULE OF THE HYPOTHETICAL SYLLOGISM.—We are, then, limited to two forms of minor premise in using the hypothetical syllogism, to wit: *The minor premise must affirm the antecedent or deny the consequent.*

Breach of the first provision in this rule results in the *fallacy of denying the antecedent* (a variety of *illicit major*); breach of the second provision in the *fallacy of affirming the consequent* (a variety of *undistributed middle*).

79. DISJUNCTIVE FORM OF CONDITIONAL SYLLOGISM.—A disjunctive proposition consists of a subject-term of which alternative predicates are asserted. Most often the predicates are two, and the disjunction is indicated by the words *either* . . . *or*. But the predicates may be three or more, when *or* is usually placed between them. Such a proposition forms the major premise, while the minor premise is affirmative or negative categorical. Contrary to the results in the ordinary syllogism, an affirmative minor premise gives a negative conclusion and a negative minor premise an affirmative conclusion. This gives rise to two moods, *that which by affirming denies* (*modus ponendo tollens*), and *that which by denying affirms* (*modus tollendo ponens*). The following are examples of these two forms:

MODUS PONENDO TOLLENS

These insects are either moths or butterflies;
But they are moths;
Therefore they are not butterflies.

MODUS TOLLENDO PONENS

These insects are either moths or butterflies;
But they are not moths;
Therefore they are butterflies.

A skilful and true disjunctive syllogism hinges upon whether the disjunction expressed in the major premise is complete. This implies, first, an enumeration of all possible alternatives, and, second, the mutual exclusiveness of the members. The best possible pairs of alternatives are, of course, logical contradictories. But the major premise is not limited to them, and, provided we are careful to choose inconsistent alternatives, our syllogism will be true. The fallacy of ignoring the principle which governs the construction of the disjunctive proposition is called *incomplete disjunction.* We should illustrate this fallacy if we argued from the major premise, *The day was either cold or rainy,* by the minor, *But it was cold,* to the conclusion, *Therefore it was not rainy.* There is no inconsistency in the day's having been *both cold and rainy.* The alternatives are not inconsistent with each other.

80. THE DILEMMA.—The dilemma (Greek, δι-, two; λῆμμα, assumption) consists in *a conditional major premise having more than one antecedent, and a minor premise which is disjunctive.* The two pairs of antecedents and their consequent or consequents make the so-called *horns of the dilemma.* It is readily seen, then, that the dilemma is really simply a combination of the two kinds of conditional proposition, hypothetical and disjunctive. If the disjunctive minor premise is affirmative, leading to an affirmative conclusion, we have the

Constructive Dilemma. If the disjunctive minor premise is negative, leading to a negative conclusion, there is the *Destructive Dilemma.* There are two varieties of constructive dilemma. The *Simple Constructive Dilemma* has for major premise two hypothetical propositions, *with the same consequent.* The *Complex Constructive Dilemma* has for major premise two hypothetical propositions, *with different consequents.* It is called complex because it leaves the conclusion disjunctive. The *Destructive Dilemma* is always complex, or otherwise it would be nothing but two disconnected destructive hypothetical syllogisms. The following symbolic formulations and examples are those used by Jevons to illustrate this type of argument:

SIMPLE CONSTRUCTIVE DILEMMA

If A is B, C is D; and if E is F, C is D;
But either A is B, or E is F;
Therefore C is D.

"*If a science furnishes useful facts, it is worthy of being cultivated; and if the study of it exercises the reasoning powers, it is worthy of being cultivated;*

"*But either a science furnishes useful facts, or its study exercises the reasoning powers;*

"*Therefore it is worthy of being cultivated.*"

COMPLEX CONSTRUCTIVE DILEMMA

If A is B, C is D; and if E is F, G is H;
But either A is B, or E is F;
Therefore either C is D, or G is H.

"*If a statesman who sees his former opinions to be wrong does not alter his course, he is guilty of deceit; and if he does alter his course, he is open to a charge of inconsistency;*

"But either he does not alter his course, or he does;

"Therefore he is either guilty of deceit, or he is open to a charge of inconsistency."

DESTRUCTIVE DILEMMA

If A is B, C is D; and if E is F, G is H;
But either C is not D, or G is not H;
Therefore either A is not B, or E is not F.

"If this man were wise, he would not speak irreverently of Scripture in jest; and if he were good, he would not do so in earnest;

"But he does it either in jest or earnest;

"Therefore he is either not wise, or not good." (*Whately.*)

Notice that in the dilemma we observe the rule of affirming the antecedent or denying the consequent.

Dilemmas are rather logical curiosities—one might almost say logical traps—than important forms of deductive argument. They are often fallacious, because, except in the case of contradictories, it is seldom that two alternatives exhaust the possibilities of the situation.

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Creighton, *An Introduction to Logic*, Ch. XI.
Welton, *Manual of Logic*, Vol. I, Bk. IV, Ch. V.
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Hibben, *Logic, Deductive and Inductive*, Pt. I, Ch. XVII.
Minto, *Logic, Inductive and Deductive*, Bk. I, Pt. IV, Ch. VII.
Jevons-Hill, *Elements of Logic* Ch. III, § VI.

REVIEW QUESTIONS

1. *Define the antecedent.*
2. *Define the consequent.*
3. *What kind of proposition forms the major premise of a hypothetical syllogism?*
4. *What kind of proposition forms the minor premise of a hypothetical syllogism?*

5. *Define a constructive hypothetical syllogism.*
6. *What does modus ponens mean?*
7. *Define a destructive hypothetical syllogism.*
8. *What does modus tollens mean?*
9. *To what form of categorical proposition does the hypothetical proposition reduce?*
10. *To what mood and figure of the categorical syllogism does the constructive hypothetical syllogism reduce?*
11. *To what mood and figure of the categorical syllogism does the destructive hypothetical syllogism reduce?*
12. *To what fallacy does denying the antecedent of an hypothetical syllogism lead? Explain in full.*
13. *To what fallacy does affirming the consequent of an hypothetical syllogism lead? Explain in full.*
14. *Commit and state the rule of the hypothetical syllogism.*
15. *What names are given to the two fallacies resulting from violation of this rule?*
16. *What is a disjunctive proposition?*
17. *What kinds of propositions form the premises of disjunctive syllogisms?*
18. *What kind of conclusions follow an affirmative minor premise and a negative minor premise, in case of the disjunctive syllogism?*
19. *What is meant by modus ponendo tollens? by modus tolendo ponens?*
20. *Upon what does the skill in constructing disjunctive syllogisms depend? What is the fallacy arising from lack of such skill?*
21. *Of what does the dilemma consist?*
22. *What is the difference between a constructive and a destructive dilemma?*
23. *What is the difference between a simple and a complex constructive dilemma?*
24. *Be prepared to express the symbolic formulations of the threc kinds of dilemma.*
25. *What rule must the dilemma conform to?*
26. *Why are dilemmas liable to be fallacious?*

EXERCISES ON CHAPTER XII

1. Make your own example of the constructive and destructive hypothetical syllogism.
2. Reduce each example to categorical form, showing the mood and figure.
3. Take the examples made in answer to Exercise 1, deny the antecedent and affirm the consequent, reduce to the categorical form, and demonstrate the fallacy.
4. Make an example of each form of disjunctive syllogism.
5. Take the example of the simple constructive dilemma given in the text, analyze it, and adapt it to the symbolic formulation.
6. Do as directed in Exercise 5 in the case of the complex constructive dilemma cited in the text.
7. Do as directed in Exercise 5 in the case of the destructive dilemma cited in the text.

CHAPTER XIII.—IRREGULAR SYLLOGISTIC FORMS

81. The Enthymeme.—Oftentimes one of the propositions forming the premises of a syllogism is a truism, that is, too obvious to any intelligent person to require expression in words. Or it may be that even the conclusion is so plain that it is tedious to put it in verbal form. *A syllogism in which a proposition is thus left to be supplied by the mind is called an enthymeme* (Greek, *ἐν, in,* and *θυμός, mind*). Most arguments met with in daily speech or reading are enthymemes. It is rare to find a fully expressed syllogism. Indeed, it would sound strange and pedantic to express the reasoning in the standard form. But whenever words like *therefore, and so, because, for, since, inasmuch as, consequently* occur, a deductive inference has taken place, and the argument, if valid, could be couched in the form of a fully expressed syllogism. It is usually the major premise which has the widest generality, and hence expresses the most obvious truth. Consequently it is this premise that is most frequently left to be supplied by the mind of the person who is following the reasoning. *A syllogism whose major premise is suppressed is known as an enthymeme of the first order.* Less often it is the minor premise which is not given. Under these circumstances we have an enthymeme *of the second order.* Occasionally even the conclusion is

not put into words. Epigrams and witty sayings are often expressed by using this device of a suppressed conclusion. This gives rise to enthymemes *of the third order.* The following are examples of the three orders of enthymemes:

ENTHYMEME OF THE FIRST ORDER

	STANDARD FORM
(* * * * * *)	(*All men must suffer;*)
All kings are men,	*All kings are men;*
And therefore must suffer.	*Therefore all kings must suffer.*

ENTHYMEME OF THE SECOND ORDER

	STANDARD FORM
All men must suffer;	*All men must suffer;*
(* * * * * *)	(*All kings are men;*)
And therefore kings suffer.	*Therefore all kings must suffer.*

ENTHYMEME OF THE THIRD ORDER

	STANDARD FORM
All men must suffer,	*All men must suffer;*
And all kings are men;	*All kings are men;*
(* * * * * *)	(*Therefore all kings must suffer.*)

It is understood that the enthymeme, since it is a regular syllogism with a proposition unexpressed, may be any valid mood in any figure in which it is valid.

82. Prosyllogisms and Episyllogisms.—Any conclusion, since it is a proposition, may become a premise of another syllogism. A syllogism whose conclusion is so used is called a *prosyllogism.* The syllogism, one of whose premises is the conclusion of another syllogism, is called an *episyllogism.* These terms are therefore relatives.

PROSYLLOGISM AND EPISYLLOGISM

No Republicans believe in free-trade;
The members of the present administration are Republicans;
Therefore the members of the present administration do not believe in free-trade;
The members of the present administration do not believe in free-trade;
The statesmen, Messrs. B. and C., believe in free-trade;
Therefore the statesmen, Messrs. B. and C., are not members of the present administration.

It is seen that the conclusion reached by E A E, first figure, in the first of the two syllogisms, is used as the major premise of the second syllogism in E A E, second figure. It is to be understood that prosyllogisms and episyllogisms may be in any valid mood.

83. THE EPICHEIREMA.—This name is given to *an episyllogism either or both of whose premises is the conclusion of an enthymeme;* if the latter, it is a double epicheirema, as in the example here added:

EPICHEIREMA

All beetles have external skeletons, for they are insects;
All fireflies are beetles, for they have hardened fore-wings;
Therefore all fireflies have external skeletons.

The syllogism itself (independent of the expressed premise of each enthymeme) is A A A in the first figure. When the enthymemes that furnish its premises are fully expressed, they also happen to fall into the mood A A A, as follows:

FIRST ENTHYMEME

(*All insects have external skeletons;*)
All beetles are insects;
Therefore all beetles have external skeletons.

SECOND ENTHYMEME

(All insects with hardened fore-wings are beetles;)
All fireflies have hardened fore-wings;
Therefore all fireflies are beetles.

Each of these enthymemes is of the first order.

84. THE SORITES.—A sorites is *a series of premises whose conclusions excepting the last are not expressed.* In other words, it is a series of prosyllogisms and episyllogisms suppressing all conclusions except the last. The word's derivation—from the Greek σωρός, *heap*—indicates quite graphically what is the type of argument. It is as though our thinking rushed on at such a headlong pace that there was no time to pause for a conclusion until the end of the process—the special conclusion wanted—is reached. The form of the sorites most usually met with is as follows:

All A is B;
All B is C;
All C is D;
Therefore all A is D.

Upon analysis this resolves itself into the following syllogisms:

FIRST SYLLOGISM

All B is C;
All A is B;
Therefore all A is C.

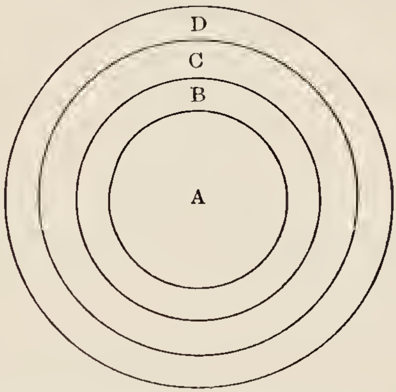
SECOND SYLLOGISM

All C is D;
All A is C;
Therefore all A is D.

An inspection of these syllogisms shows us that *no premise but the first may be particular.* For if B, which is the middle term in the first syllogism above, and also in the first two premises of the sorites, were in a particular premise in the case where it has the position of subject-

term, there would be no distribution of the middle term. Of course it cannot be distributed as the predicate of an affirmative proposition (in the first premise), so it must be distributed as the subject of the second premise. Hence this premise must be universal (the I and O do not distribute their subjects). What is said of B holds good of C and any other middle terms that are used in the course of the argument.

Likewise *no premise but the last may be negative.* For if any other were negative, it would make the con-



clusion negative; hence its major term would be distributed. But such distribution could be provided for only if the last premise (where the major term occurs in the sorites) were negative.

In this arrangement of the sorites the subject of the first premise is the subject of the conclusion, and the predicate of the last premise is the predicate of the conclusion. It is commonly called the Aristotelian sorites. It can be graphically represented by a number of concentric circles, the smallest representing the denotation of A, the next that of B, the next that of C, etc.

The following is a concrete example of the sorites:

ARISTOTELIAN SORITES

All whales are mammals;
All mammals are warm-blooded;
All warm-blooded animals are vertebrates;
All vertebrates have a spinal-cord;
Therefore all whales have a spinal-cord.

There is another form of the sorites called the Goclenian,[1] in which the subject of the last premise and the predicate of the first are united in the conclusion. Schematically represented, it is as follows:

All A is B;
All C is A;
All D is C;
Therefore all D is B.

Expanded into syllogisms this series of arguments takes the following form:

FIRST SYLLOGISM

All A is B;
All C is A;
Therefore all C is B.

SECOND SYLLOGISM

All C is B;
All D is C;
Therefore all D is B.

It will be remarked that the suppressed conclusions form the minor premises of the following syllogism in the Aristotelian sorites, whereas in the Goclenian they form the major premise.

The rule respecting the character of the premises is the reverse of that given for the Aristotelian form. In the Goclenian sorites *only the first premise may be*

[1] After Rudolf Goclenius (1547–1628), who invented it.

negative and only the last particular. If the second or any other than the first were negative we should have a negative conclusion; hence a distributed major term. But if the second (for instance) were negative the first would have to be affirmative (Rule of the Syllogism 1, II); hence the major term would not be distributed in it (because in the predicate position); hence there would be illicit major.

Were any but the last premise of the Goclenian sorites particular we should have undistributed middle. For the middle term would be the subject in one premise of a particular proposition, and hence undistributed there; and the predicate in the other premise of an A proposition (Rule of the Syllogism 2, II prevents two particulars, and the first premise is the only one that may be negative; hence we are limited to A), and hence undistributed there. The following is a concrete example of the Goclenian sorites:

GOCLENIAN SORITES

All vertebrates have a spinal cord;
All warm-blooded animals are vertebrates;
All mammals are warm-blooded;
All whales are mammals;
Therefore all whales have a spinal cord.

REFERENCES

Creighton, *An Introductory Logic,* Ch. X.
Jevons-Hill, *Elements of Logic,* Ch. III, § V.
Welton, *Manual of Logic,* Vol. I, Bk. IV, Ch. VI.
Bradley, *The Principles of Logic,* pp. 348–360.
Hibben, *Logic, Deductive and Inductive,* Pt. I, Ch. XVIII.

REVIEW QUESTIONS

1. *Define enthymeme and give its derivation.*
2. *Which premise is most frequently suppressed, why, and what is such an enthymeme called?*
3. *What is an enthymeme of the second order? of the third order?*
4. *What is a prosyllogism? an episyllogism?*
5. *Define epicheirema.*
6. *Define sorites; Aristotelian sorites; Goclenian sorites.*
7. *What rule governs the Aristotelian sorites? Be prepared to demonstrate both parts of this rule.*
8. *What rule governs the Goclenian sorites? Be prepared to demonstrate both parts of this rule.*
9. *What terms form the subject and predicate of the conclusion in each kind of sorites.*
10. *Which premise does the suppressed conclusion make in each form of sorites?*

EXERCISES ON CHAPTER XIII

1. Construct an enthymeme of each of the three orders.
2. Make a prosyllogism and an episyllogism.
3. Make an epicheirema.
4. Make an Aristotelian sorites and then rearrange it in the Goclenian form.

CHAPTER XIV.—DEDUCTIVE FALLACIES

85. Classification of the Deductive Fallacies. —*Fallacy is the generic term applied to errors in the process of reasoning, whether inductive or deductive, and especially in syllogistic reasoning.*

While engaged upon the deductive syllogism we noticed certain tendencies toward formal errors in the construction of arguments. We shall now name all the more common errors incidental to deduction, and shall describe more particularly those errors that lie not in the form but in the subject-matter of deductive reasoning.

No classification of such errors can be perfectly satisfactory or scientific, because many of the types are extremely subtle and are easily confounded. Again, many fallacious arguments are intricate and complex almost beyond the power of analysis, and really involve several fallacies. Yet some sort of classification, even though it be only tentative, will aid us in comprehending this difficult and important chapter in logic.

The subject of fallacies is important because they are so insidious and common. Particularly where wordy debates take the place of modest and careful statements of the reasons for and against a position, we find fallacies lurking. And since proneness to talk much and say little is a universal human failing, we may readily see how many are the opportunities for fallacy. It is to make the student feel the danger and keep his attention

alert to prevent these errors that a section of our general subject is assigned to this topic.

Following time-honored custom, we distinguish two great kinds of deductive fallacy: first, those due to incorrectness in the *syllogistic form,* and second, those resulting from a misapprehension of the content or subject-matter of the argument. The first kind are technically called *logical* or *formal,* and the second kind *material fallacies.* The formal fallacies will be given a place in the classification, but they have already been sufficiently treated in discussing the limitations of the form of the syllogism in connection with moods and figures. The material fallacies will be more fully explained in this chapter.

Any breach of a rule of the syllogism causes a formal fallacy. They were considered in chapters X, XI, XII, and XIII. They are as follows: (1) *Four terms;* (2) *Undistributed middle;* (3) *Illicit process of the minor term;* (4) *Illicit process of the major term;* (5) *Conclusion from negative premises;* (6) *Conclusion from particular premises;* (7) *Affirmative conclusion with a negative premise;* (8) *Universal conclusion with a particular premise;* (9) *Negative conclusion without a negative premise;* (10) *Denying the antecedent of a hypothetical syllogism;* (11) *Affirming the consequent of a hypothetical syllogism;* (12) *Imperfect disjunction in the major premise of a disjunctive syllogism;* (13) *Any other premise than the first in the Aristotelian and the last in the Goclenian sorites particular;* (14) *Any other premise than the last in the Aristotelian and the first in the Goclenian sorites negative.*

The material fallacies may be divided into two classes, one including those that can be attributed to ambiguity, the other those that have their origin in assuming unconsciously the issue to be determined by the course of the argument. The former species may be called the *Fallacies of Ambiguity*, the latter the *Fallacies of Unwarranted Assumption.*

MATERIAL FALLACIES

Fallacies of Ambiguity

1. Equivocation.
   - (a) Ambiguous major term.
   - (b) Ambiguous minor term.
   - (c) Ambiguous middle term.
2. Amphiboly (Amphibology).
3. Ambiguity between collective and distributive use of terms.
   - (a) Composition.
   - (b) Division.
4. Accent.
5. Figure of speech.

Fallacies of Unwarranted Assumption

1. Confusion of the general and the exceptional.
   - (a) Accident.
   - (b) Converse accident.
2. Irrelevant conclusion (*Ignoratio elenchi*).
   - (a) *Argumentum ad hominem.*
   - (b) *Argumentum ad populum.*
   - (c) *Argumentum ad ignorantiam.*
   - (d) *Argumentum ad vericundiam.*
3. Begging the question (*Petitio principii*).
4. Consequent (*Non sequitur*).
5. False cause (*Non causa pro causa*).

86. FALLACIES OF AMBIGUITY.—Ambiguity of terms has already been discussed at some length in chapter III. When a term readily suggests more than one meaning, it is ambiguous, and occasions the fallacy of

equivocation. When such a term is used in the syllogism it may be employed as major, minor, or middle term. Hence there are three syllogistic forms of this fallacy, namely, ambiguous major, where the ambiguous term occurs in the major premise and as predicate of the conclusion, ambiguous minor, where the ambiguous term is found in the minor premise and as subject of the conclusion, and ambiguous middle, where the ambiguous term is found only in the premises. The fallacy of equivocation is nearly always found in the form of ambiguous middle. An example of it is as follows:

Evil conditions deserve moral condemnation;
Headaches are evil conditions;
Therefore headaches deserve moral condemnation.

Here the fallacy lies in two different meanings conveyed by the middle term, *evil conditions*. In the major premise it means *conditions of moral evil*, and in the minor, *conditions of physical evil.*

Another example is as follows:

Bloody deeds should be severely punished by law;
Butchering animals for market is a bloody deed;
Therefore butchering animals for market should be severely punished by law.

The student can here detect the fallacy for himself. The more serious cases of equivocation are to be found in long arguments where the difference in meaning of the terms on the two occasions of their use is more or less disguised under the veil of complex construction.

Amphiboly is another form of ambiguity. It is also known under the longer name of *amphibology*. It is an

error of grammatical construction that admits of a double interpretation. A classical example is furnished by a line in Shakespeare's *Henry VI: "The Duke yet lives that Henry shall depose."* This may mean either that Henry is to depose the Duke or that the Duke is to depose Henry. Ludicrous fallacies of this character are often found in the want-advertisements of newspapers, such as: *A piano wanted by a lady with a rich tone and a pianola attachment; A second-hand morris-chair is wanted by a bachelor with richly carved claw-feet.*

Many words may be used both collectively and distributively. This fact gives rise to a much more serious form of ambiguity than that last discussed. There are two phases of the fallacy, the first called *composition* and the second *division.* Composition, as the name itself indicates, consists in arguing from a distributive meaning of a term in the premises to a collective meaning in the conclusion. Division consists in arguing from a collective meaning in the premises to a distributive meaning in the conclusion. A classical illustration of composition is furnished in the following argument:

All (i. e., any of) the angles formed at the point of intersection of two straight lines are less than two right angles;

Angles A E B, B E C, C E D, and D E A are the angles so formed;

Therefore all these angles (together) are less than two right angles.

The same fallacy is found in the following argument: *Because all the acts (any act) of a man's career may be mistakes, therefore his whole career (the sum-total of acts) must be a failure.* Or again: *Since all men are likely to*

have erroneous views on matters of public policy, therefore democracy must result in poor government. Granted that any particular man is liable to wrong views, it is not correct to argue that the views of all together will be wrong.

Mill, the celebrated English logician, is himself guilty of this fallacy in a frequently cited illustration found in his work on *Utilitarianism* (p. 53):

> *No reason can be given why general happiness is desirable except that each person, as far as he believes it to be attainable, desires his own happiness. This, however, being a fact, we have not only all the proof which the case admits of, but all which it is possible to require, that happiness is a good; that each person's happiness is a good to that person, and the general happiness, therefore, a good to the aggregate of all persons.*

A peculiar form of composition seems to lie at the root of many popular notions of the functions and capacities of the State. The imagination frames a kind of superhuman agency, and invests it with the aggregate wisdom and power of its component individuals, whereas, in fact, its limits are determined more nearly in accordance with the political sagacity of the single ablest and most experienced statesman.

The following syllogism illustrates the fallacy of division:

> *All the angles (together) formed at the point of intersection of two straight lines are equal to four right angles;*
> *Angle A E B is one of the angles so formed;*
> *Therefore angle A E B is equal to four right angles.*

We detect more subtle instances of division in popular views on social and economic subjects.

For example, the public suffers a grave injustice at the hands of a public-utilities corporation. The power

of committing the injustice and the responsibility therefor are conditioned upon a pool of capital and interests the full issue of which no individual member of the corporation can foretell. But popular judgment distributes the sum-total of the opprobrium that is felt against the "trust" as a collective agency to each capitalist composing it.

Again, the socialist is easily beguiled with the dream of a Utopia where poverty and misery disappear. His mind dwells upon the pleasing picture of the nation's aggregate wealth, then applies this total wealth to the relief of individual distress, and by this legerdemain of logical division creates a society in which penury is unknown. In the socialistic state every one is well provided, just because each and all are the beneficiaries of the total wealth of the State.

The relation between composition and division is very skilfully shown by Welton [1]:

> *The most common form of the fallacy may be reduced to an implicit confusion between a disjunctive and a copulative proposition. Thus, the spendthrift, falling into the fallacy of composition, argues "I can afford a or b or c or . . . z, therefore I can afford a and b and c and . . . z." On the other hand, the converse fallacy of division is often found lurking in the argument by which a miserly person refuses to subscribe to any charitable object. "I cannot afford to subscribe to a and b and c and . . . z, therefore I cannot afford to subscribe to a or b or c or . . . z."*

The word *all* is peculiarly misleading and liable to fallacious interpretation. It may mean *each and every* or it may mean *the sum-total.* The best test to apply in each instance of doubt is to decide which it means before going ahead with the inference.

[1] *Manual of Logic,* Vol. II, p. 247.

Accent, as the name sufficiently indicates, is a fallacy *occasioned by misleading accent or special emphasis.* Jevons quotes a laughable instance from the Bible, *First Book of Kings*, Ch. XIII, verse 27: "And he spake to his sons, saying, Saddle me an ass. And they saddled *him.*" The italics are used in the verse to show that the translators supplied the word *him.* But it might easily be mistaken to mean that they saddled the prophet referred to for the ass. Almost any of the Ten Commandments may be made to show this fallacy by emphasizing the word *thou,* leading to the implication that all others may do what the command prohibits, but that the imaginary person addressed must not. In Shakespeare's *Macbeth* there occurs an important line put into the mouth of Lady Macbeth, which lends itself very readily to the fallacy of accent. It is in Act I, Scene vii, where Lady Macbeth is trying to arouse the flagging courage of her husband. The dialogue is as follows:

Macbeth. If we should fail?
Lady Macbeth. We fail!

The punctuation given here is that of the Temple Edition, from which the quotation was made. Lady Macbeth's answer seems capable of at least three interpretations, and these will be indicated by different modes of print and punctuation, and then explained.

Macbeth. If we should fail?
Lady Macbeth. *We* fail?

Here the meaning would be pretended surprise at the absurd idea of failure when the enterprise was in the hands of such a pair.

Macbeth. If we should fail?
Lady Macbeth. We fail?

Here the line would evidently be meant by the author as a repetition of Macbeth's question as a preliminary to showing him that a little courage is all that is needed to prevent failure.

In addition to these there is the form first given. Here the author's intention would seem to be to convey a hint of that fatalism that often characterizes woman when great and hazardous enterprises are at stake, and that certainly marks Lady Macbeth's moral attitude at other climaxes of the play. *If we fail, we fail, and that's an end of it,* would be the meaning, if this interpretation were taken.

This fallacy is committed whenever an excerpt is so taken from a letter or book as to seem to have a meaning different from that which it would have with the context. Many controversies result from taking extracts from books and contending that they mean something different from the author's evident intention when the passage is read with its context. Italics may be used in the same way to cause a wrong impression of the author's meaning. Fair-minded parties to a published controversy always say whether the italics are their own or the author's they are combating when they use this form of printer's device for bringing out emphasis.

The Fallacy of Figure of Speech is due to a confusion between parts of speech. It is so trivial in character as to deserve mention only for the sake of completeness. Occasionally, however, a serious instance of this fallacy occurs. Mill, in the book quoted from in illustration of *composition,* falls into *figure of speech* when he argues: "*The only proof capable of being given that an object is visible is that people actually see it. The only proof*

that a sound is audible is that people hear it. . . . In like manner, I apprehend, the sole evidence it is possible to produce that anything is desirable is that people actually desire it." (*Utilitarianism,* pp. 52–53).

In this case the fallacy lies in the writer's failure to distinguish between the two meanings of the suffix *-ible* (*-able*). In the words *visible* and *audible* it has the force, *capable of being;* in *desirable,* on the other hand, its force is *fit to be.* To argue through this confusion that because an object's visibility is proved in the fact of seeing it, therefore a thing's desirability is proved by the fact that it is desired, leads to the utter rout of clear thinking.

87. Fallacies of Unwarranted Assumption.—These fallacies have the common characteristic of the false and surreptitious assumption of some phase of the matter to be proved. Such assumption may be the entire conclusion assumed when presented in different verbal form. On the other hand, it may be the assumption of some seemingly trivial aspect of the question at issue. We are quite ready, in the heat of debate, to take the point as proved when it is only assumed. Much the most serious of the fallacies arise in this way.

The Fallacy of Accident and its converse arise from a confusion of the general and the exceptional. A moment's thought teaches us that every inductive generalization, whether it be concept, law, or principle, depends upon abstraction from the concrete form in which our experiences present themselves. In classification, for instance, the attention is focussed upon the attributes that are common to all the particulars involved, and ignores for the moment the manifold ways

in which these same particulars manifest their individuality. Now there can be no doubt that such a process, necessary though it is, does a certain violence to the real nature of things. This is seen in philosophy, which is a science of first and hence most general principles. It is charged with some justice with keeping aloof from actual experience, and hence with inadequacy in the performance of its function of systematizing and coordinating knowledge. On the other hand, the common mind, which is boastful of not philosophizing, fails of just those generalizations that are so essential to a comprehension of things in their larger meanings. The plain and prosaic mind, the plodding soul, the gatherer of endless particulars, never attains a sweep of the eye over the whole universe of experience, never takes a world-view, apprehends his environment as the worm does, by groping from bit to bit rather than with the majestic outlook of the eagle. Here we have two extremes. Neither gets the true insight. As Kant long ago said, *General notions without contents are empty, while sense-impressions without concepts are blind.*[1] Truth lies in that activity of intellect which apprehends the general as existent amid the particular, but at the same time keeps a firm grip upon the particulars out of which the general has originated. Any violation of the safe middle course leads to the fallacy now under consideration. If the mind retains such an impression of the general as to ignore wherein the particular departs from it, we have the fallacy of accident. If the mind loses itself in particulars, so that it cannot see the forest

[1] See *Critique of Pure Reason*, Muller's tr., 2d ed., p. 41.

for the trees, and thus fails to get the universal out of the particular, there occurs the converse fallacy of accident. Now the general may be regarded as that which holds true of all the members of a class, whereas the particular includes both the general attributes and the special ways in which each individual departs from the common attributes. Hence the particular becomes in its second aspect the peculiar, special, or accidental. It is therefore commonly said that the fallacy of direct accident confuses the general with the accidental; that of converse accident confuses the accidental or special with the general. But, after all, this means no more than to say that the limits of the field of attention are so narrow as now to exclude the special and again to exclude the general.

The following examples of this fallacy will serve as illustrations:

It would be the direct fallacy of accident to argue that because it is right to speak the truth, it is therefore right to tell an acquaintance just what our private judgment of him is; that because it is right to deal fairly with all with whom we do business, it is therefore necessary that we merely reimburse ourselves at a moderate rate of interest for the capital invested when we sell a piece of property. In line with the same fallacious argument is the common opinion that one should voice his honest convictions upon a political or religious question whenever opportunity offers, on the ground that every one should form and express strong convictions on fundamental problems. Advocates of higher education fall into this fallacy when they argue that, because educa-

tion results in general improvement, every one, irrespective of ability or circumstances, should be given the advantages of a college education. The sum and substance of the matter is that there are exceptions to every rule, and it is the part of clear thinking to recognize the places where the rule will not apply. On the other hand, the claim of the self-made man that since he succeeded without a college education, it is a pure waste of time to spend four years in college, is the fallacy in the converse form.

The converse fallacy is illustrated also in the conclusion that tact and diplomacy are required in every situation just because they are demanded in some situations. Many an habitual prevaricator has been made because it was necessary under some peculiar stress of circumstances for him to take one step in deception. When, some years ago, Professor Atwater asserted as the result of experiments that alcohol is a food, a storm of protest arose among ardent and zealous teetotalers. Their reasoning, apart from its quality from a moral point of view, was as a bit of logic extremely bad. For, in effect, it was that the admission that alcohol is a food, under certain peculiar circumstances, was tantamount to recommending it as a common article of diet—manifestly an example of converse accident.

This topic may be brought to a close with an extremely sane comment upon the intellectual and moral consequences of this kind of erroneous thinking from Lotze's *Logic:*

Two general modes of fallacious thought are developed by the habitual commission of these fallacies, and illustrate them on a

grand scale. The first is doctrinairism, the second narrow-mindedness. The doctrinaire is an idealist who refuses to see that though ideas may be right in the abstract, yet the nature of the circumstances under which and of the objects to which they are to be applied must limit not only their practicability but even their binding force. The narrow-minded, on the other hand, can recognize and esteem no truth and no ideal, even the most universally valid, except in that special form to which they have become accustomed within a limited circle of thought and personal observation. Life is a school which corrects these habits of mind. The parochially minded man sees things persist in spite of himself in taking shapes which he considers unprecedented, but he finds the world somehow survives it, and learns at last that a system of life may be excellent and precious, but that it is rash from that to argue that it is the only proper mode of orderly existence. And the enthusiast for ideals, when he sees the curtailment which every attempt at realization inflicts on them, learns the lesson which the disjunctive theorem might have taught him." [1]

There are several forms of the *Fallacy of Irrelevant Conclusion*. Early logicians classed them together under the somewhat misleading caption, *Ignoratio Elenchi* (*ignorance of the refutation*). In fact, these several types of fallacy are the great devices of sophistical argument; and so, as a rule, they are employed with full consciousness of their power to mislead. They are the tools of the wily and sophisticated against the ignorant and unthinking. In all the forms there is a strong appeal to the feelings, prejudices, and passions of those to whom the argument is addressed. Now it is a general rule that the feelings are antagonistic to good logic. Feeling may carry us away in a flood of outgoing nervous energy that results in powerful and

[1] Vol. II, p. 5, Eng. trans. (Quoted from Hibben's *Logic*.)

masterful actions, but it rarely makes for clearness of intellectual vision. Hence any appeal that an argument makes to the feelings should be regarded with suspicion. But special pleaders, whose end is the practical one of winning support for a cause, care little for the logic of the situation if only they can arouse in their hearers the conviction that assures a vigorous line of action. Therefore they employ any expedient that will superinduce belief in their cause.

Argumentum ad hominem is that form of irrelevant conclusion in which the speaker assails the general character of the person against whom his tirade is directed instead of proving the actual charge that is being made against him. It is often used by lawyers whose cases are weak against a witness whose testimony is unwelcome. Discrediting a witness is a regular part of the business of an attorney. It is perfectly legitimate so long as it is confined to the credibility of the testimony and strikes at the point at issue. But when a lawyer seeks to weaken the effect of a witness's testimony by allusion to some derogatory circumstance that does not bear upon the case he is guilty of *argumentum ad hominem*. Striking instances are afforded also whenever a lawyer, in pleading before a jury, assails the general character of the defendant, not with the purpose of showing that his previous record tends to sustain the charge against him, but in order to arouse in the jury's mind a prejudicial feeling. Charges of inconsistency urged by one political candidate against another are usually cases of *argumentum ad hominem*. They aim to browbeat and confuse the opponent, and to lead to

his utter rout and undoing in the minds of the thoughtless many who assume in their leaders a kind of superhuman wisdom, and hencc revolt at any suggestion of changed opinion. This kind of fallacy is often ludicrously shown in the social ostracism visited against persons following certain undignified occupations. He cannot be a good man to know because he is the hangman is the type of thinking here alluded to. She cannot be admitted to our select circle because she lives on Nowhere Street.

Argumentum ad populum is an appeal to the passions and prejudices of those to whom it is addressed rather than to their reasoning powers. It is the great weapon of demagogues. Almost all political speeches contain at least some instances of it, and many are almost entirely constructed out of it. It is easy to move the feelings of a person but hard to convince his intellect. Then there is also the fact that men are easily aroused to passion *en masse* where the same appeal would not stir them as isolated individuals. During the French Revolution the cry, "*En bas la noblesse!*" was sufficient to move a mob of street loiterers to the highest pitch of frenzy that stopped at nothing short of the utter annihilation of its unhappy victim. This is an instance of this terrible fallacy of the demagogue and of what it can do. Another instance, the best in literature, is Antony's speech to the Roman mob in Shakespeare's *Julius Cæsar.* The whole of that speech is a departure from the question at issue between Brutus and Cæsar's adherents. But, nevertheless, Mark Antony plays upon the passions and cupidity of those street vagabonds

with such demagogic skill that he turns them from enemies into slaves, ready to do his bidding, and all with scarcely an argument addressed to the intellect. The following excerpts from the speech show the *argumentum ad populum* put to its use in a wonderfully skilful manner.

Antony.—But here's a parchment with the seal of Cæsar;
I found it in his closet; 'tis his will:
Let but the commons hear this testament—
Which, pardon me, I do not mean to read—
And they would go and kiss dead Cæsar's wounds
And dip their napkins in his sacred blood,
Yea, beg a hair of him for memory,
And, dying, mention it within their wills
Bequeathing it as a rich legacy
Unto their issue.
Fourth Citizen.—We'll hear the will: read it, Mark Antony.
All.—The will, the will! we will hear Cæsar's will.
Antony.—Have patience, gentle friends, I must not read it;
It is not meet you know how Cæsar loved you.
You are not wood, you are not stones, but men;
And, being men, hearing the will of Cæsar,
It will inflame you, it will make you mad:
'Tis good you know not that you are his heirs;
For if you should, O, what would come of it!
Fourth Citizen.—Read the will; we'll hear it, Antony;
You shall read us the will, Cæsar's will.

* * * * * * * * * *

Antony.—You will compel me then to read the will?
Then make a ring about the corpse of Cæsar,
And let me show you him that made the will.

* * * * * * * * * *

Second Citizen.—Room for Antony, most noble Antony.

* * * * * * * * * *

Antony.—If you have tears, prepare to shed them now.
You all do know this mantle: I remember
The first time ever Cæsar put it on;

'Twas on a summer's evening, in his tent,
That day he overcame the Nervii:
Look, in this place ran Cassius' dagger through:
See what a rent the envious Casca made:
Through this the well-belovèd Brutus stabb'd;
And as he pluck'd his cursed steel away,
Mark how the blood of Cæsar follow'd it,
As rushing out-of-doors, to be resolved
If Brutus so unkindly knock'd, or no:
For Brutus, as you know, was Cæsar's angel:
Judge, O you gods, how dearly Cæsar loved him!
This was the most unkindest cut of all;
For when the noble Cæsar saw him stab,
Ingratitude, more strong than traitors' arms,
Quite vanquish'd him: then burst his mighty heart;
And in his mantle muffling up his face,
Even at the base of Pompey's statue,
Which all the while ran blood, great Cæsar fell
O, what a fall was there, my countrymen!
Then I, and you, and all of us fell down,
Whilst bloody treason flourish'd over us.
O, now you weep, and I perceive you feel
The dint of pity: these are gracious drops.
Kind souls, what weep you when you but behold
Our Cæsar's vesture wounded? Look you here,
Here is himself, marr'd, as you see, with traitors.

First Citizen.—O piteous spectacle!
Second Citizen.—O noble Cæsar!
Third Citizen.—O woful day!
Fourth Citizen.—O traitors, villains!
First Citizen.—O most bloody sight!
Second Citizen.—We will be revenged.
All.—Revenge! About! Seek! Burn! Fire! Kill!
Slay! Let not a traitor live!

* * * * * * * * * *

Antony.—Why, friends, you go to do you know not what:
Wherein hath Cæsar thus deserved your loves?
Alas, you know not; I must tell you then:
You have forgot the will I told you of.

* * * * * * * * * *

Here is the will, and under Cæsar's seal.
To every Roman citizen he gives,
To every several man, seventy-five drachmas.
* * * * * * * * * *
Moreover, he hath left you all his walks,
His private arbors and new-planted orchards,
On this side Tiber; he hath left them you,
And to your heirs forever; common pleasures,
To walk abroad and recreate yourselves.
Here was a Cæsar! when comes such another?
First Citizen.—Never, never. Come, away, away!
* * * * * * * * * *
Second Citizen.—Go fetch fire.
Third Citizen.—Pluck down benches.
Fourth Citizen.—Pluck down forms, windows, anything.
Antony (*alone*).—Now let it work. Mischief, thou art afoot,
Take thou what course thou wilt.

The form of Irrelevant Conclusion known as *argumentum ad ignorantiam* consists in throwing the burden of proof upon the other party to an argument when, in the nature of the case, disproof is as impossible as proof. In other words, it consists in maintaining a position on the flimsy pretext that the opposite cannot be shown to be true. A man who contends that there are ghosts just because nobody has been able to prove positively that there are not is guilty of this form of fallacy. If we were to give credence to every merely possible form of existence, our universe would be teeming with all sorts of bizarre forms. Much that is not truly contradictory to the fundamental laws of thought *might* exist, so far as mere formal possibility is concerned, but in actuality *does not exist.* To hold that a thing is true if you can't prove it untrue is therefore much like enriching yourself by taking from one pocket and putting into the other.

Argumentum ad vericundiam, another form of Irrelevant Conclusion, consists in enlisting a great and reputable name in support of a position in place of proving it on rational grounds. It is in line with the universal appeal to authority that characterized the thinking of the Middle Ages and produced such barren results. It may even reach such extremes that a person who makes a great reputation in one field of activity is cited as authority on other matters about which he has no special knowledge. The use of books may lead to a sort of habitual fallacy of *argumentum ad vericundiam.* Some people fall into a way of attaching a superstitious reverence to cold, black print, and unthinkingly believe everything that they see in such form, without bringing it into accord with their own experience by a little hard thinking. It is of this class, the slaves of borrowed opinion, that Locke speaks in his *Conduct of the Understanding:*

> *Reading furnishes the mind only with materials of knowledge; it is thinking makes what we read ours. We are of the ruminating kind, and it is not enough to cram ourselves with a great load of recollections; unless we chew them over again they will not give us strength and nourishment. . . . The memory may be stored, but the judgment is little better, and the stock of knowledge not increased by being able to repeat what others have said or produce the arguments we have found in them. Such knowledge as this is but knowledge by hearsay, and the ostentation of it is at best but talking by rote and very often upon weak and wrong principles. For all that is found in books is not built upon true foundations, nor always rightly deduced from the principles it is pretended to be built on.*[1]

[1] *Conduct of the Understanding,* Bohn ed. of Locke's *Works,* Vol. I, p. 64.

Begging the question (*petitio principii*) is among the most common of the fallacies. It consists in assuming the point to be proved as one of the premises—usually, of course, couched in different terms. Another name for the fallacy is *circulus in probando, i. e.*, a circle in the proof. As just hinted, this form of fallacy would be detected, even by the unthinking, if the phraseology of the assumed premise were identical with that of the ostensible conclusion. Hence the conclusion is usually disguised in synonymous but seemingly different terms. The fact that our language is derived from two great sources, Latin and Anglo-Saxon, greatly facilitates the ease with which we commit this fallacy. Jevons cites a good instance in the following spurious argument: "*Consciousness must be immediate cognition of an object; for I cannot be said really to know a thing unless my mind has been affected by the thing itself.*" [1] The conclusion here is, *Consciousness is immediate cognition of an object;* the premise supporting it is, *All instances of knowing a thing are cases of the mind's being affected by the thing itself.* A little analysis shows that the term *knowing a thing* has here an identical meaning with the term *consciousness,* while the phrase *the mind's being affected by the thing itself* can have no other meaning than *immediate cognition of an object.* The following argument is another case of begging the question: *I support the conclusion that the soul is immortal upon the evident ground that the rational element in human nature does not perish.*

In this connection we must pay a moment's attention to "question-begging epithets," a subtle form of the

[1] *Elements of Logic,* p. 174.

fallacy of *petitio principii.* If a voter reasons that a given candidate for office is a demagogue, and hence should not receive his support, he is begging the question by using that term of reproach, unless he is certain of his grounds for the assertion; and as a rule such certainty can only be assured by the candidate's election to office. For only then can the speciousness of his ante-election promises be shown up against him. In other words, you can *prove* demagoguery only by testing a man with the administration of power. Most of the calumniations of "mud-slinging" political campaigns are in reality simply question-begging epithets, the real proof of the charge being very rarely adduced. The authors of campaign canards, who are usually practical students of human nature, know well enough that a mere hint is confounded with proof in the minds of the unthinking multitude.

The fallacy of *non sequitur* or *consequent* includes all extremely loose reasoning where the conclusion has no real relation whatsoever to the premises. Many arguments that are accepted by audiences as cogent reasonings are really *non sequitur.* In following a train of reasoning off-hand attention is quite likely to focus upon the alleged conclusion. Even the word *therefore,* owing to association of ideas, prepares the mind to accept as proved the statement which it introduces. Hence, if a certain parade of reasoning be made, and the dignity of long words be added in support of the pretence, a listener who has not the opportunity to analyze the argument will often give his assent even though the conclusion bears no real kinship to the prem-

ises. The following would be an example: *Taxation without the knowledge of those who are taxed is undemocratic; indirect taxes are unwittingly paid by the tenant; hence the landlord has no right to collect the rent.*

The fallacy of *false cause*, sometimes named by the Latin phrases *non causa pro causa* and *post hoc ergo propter hoc*, while usually listed among the deductive fallacies, seems to belong more properly among those incident to induction. The farmer commits it when he maintains that the weather is due to the phases of the moon; the astrologer commits it when he claims that our destinies are controlled by the stars; and in general it is committed whenever mere sequence in time, noted on a few occasions, is taken to mean causal connection. This fallacy is a frequent incident of superstitious interpretations of nature and human experience. Savages and the semi-civilized fall easy victims to the delusion that all sorts of casual occurrences are causally connected. It is only with the lessons of larger experience that most of these chance connections are seen to be purely haphazard in character and do not imply necessary connection of cause and effect. Even among people who are advanced enough by universal education to think for themselves, it is not at all uncommon to hear a causal relation attributed to events that show merely chance concomitance. Indeed, national elections in the United States almost invariably go against a party that has had the misfortune to be in control during a period of financial depression, though it may be doubted whether governmental policy has ever been directly responsible for hard times in this country.

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REVIEW QUESTIONS

1. *What kinds of fallacies have been studied in connection with the syllogism?*
2. *Why does classification of the fallacies present special difficulty?*
3. *What are the two great kinds of deductive fallacies?*
4. *What are the two classes of material fallacies?*
5. *Explain ambiguous middle.*
6. *What is amphiboly?*
7. *Explain composition and division.*
8. *What do you understand by the fallacy of accent? How may it be used in quoting from an author?*
9. *What is common to all the fallacics of unwarranted assumption?*
10. *Explain the fallacy of accident and account for its origin. The same for the converse fallacy of accident.*
11. *What is the moral pointed out in the quotation from Lotze given on pp.* 177–178?
12. *What factor is common to the several fallacies of irrelevant conclusion? Who is especially likely to make use of them?*
13. *What is the nature of thc argumentum ad hominem?*
14. *Explain the fallacy of argumentum ad populum.*
15. *What do you understand by the argumentum ad ignorantiam?*
16. *What is the argumentum ad vericundiam? Why is rapid reading liablc to occasion it?*

17. *Explain petitio principii. To what circumstance is it usually due? How does language assist it?*

18. *What do you understand by question-begging epithets?*

19. *Explain non sequitur.*

20. *Is false cause inductive or deductive? What is its origin?*

EXERCISES ON CHAPTER XIV

1. Make an example of ambiguous middle.

2. Make two or three examples of amphiboly. Also consult the personal or want advertisement columns of a daily paper and get examples.

3. Make an example each of composition and division.

4. Make an example of the fallacy of accent. Illustrate how it might occur in quoting from a letter or book.

5. Exemplify the fallacy of accident. Also its converse.

6. Imagine yourself defending a weak case at law and show how you might resort to *argumentum ad hominem.*

7. Imagine yourself addressing a public gathering on some political topic and construct a fitting *argumentum ad populum.*

8. Examine the part of Antony's speech cited in the text, make a list of the *argumenta ad populum,* and then cite the evidence which shows the effect upon the mob.

9. Show an instance where *argumentum ad ignorantiam* might be used.

10. Imagine or instance from your reading or other experience a case of *argumentum ad vericundiam.*

11. Make an argument (spurious) illustrating *petitio principii.* Also give an instance of question-begging epithet.

12. Prepare an instance of *non sequitur.*

13. Present an instance of false cause.

Additional exercises on the deductive syllogism and its fallacies (referring particularly to topics treated in chapters X, XI, XII, XIII, and XIV).

The following arguments are not stated in syllogistic form. Some of them are valid and others are fallacious. The student is to restate them in the simplest propositional form, and to arrange them as formal syllogisms. Then he is to determine

their mood and figure if valid, and to determine the kind and position of the fallacy if invalid. If any should prove to be irregular syllogistic forms, reconstruct them as regular syllogisms, supplying the missing propositions. In working out these exercises the student is cautioned to remember that the conclusion is usually introduced by such words as *therefore, hence, consequently,* or other signs of inference; while the premises (if stated after the conclusion) are often intodruced by *for, because, since, etc.* Remember also to work from the definitely determined position of the terms in the conclusion back to the premises, bearing in mind that the subject of the conclusion is the minor term, its predicate the major term, and that the premise containing the former is the minor premise, while that containing the latter is the major premise.

EXERCISES

1. The soul must be immortal, for it cannot be proved that it is not.
2. "It was not on account of family that (he) was snubbed by the British nobility after he had won the Derby by not being invited to the King's dinner."
3. Impractical men are sometimes highly educated; hence this man must be impractical, for he is highly educated.
4. All material bodies impress the senses; mind does not impress the senses. What is the inference?
5. If this cashier used the depositors' money to speculate with, he should be sent to prison; he did not use it, therefore he should not be sent to prison.
6. If this cashier used the depositors' money to speculate with, he should be sent to prison; he is under sentence, therefore he used it.
7. All beetles are insects, for both insects and beetles have bodies divided into head, thorax, and abdomen.
8. All beetles are insects, and hence have bodies divided into head, thorax, and abdomen.
9. A college education does not pay, for most self-made Americans have succeeded without it.
10. There is no use in being honest, for many dishonest men make a fortune.

11. I will not call the conductor's attention to the fact that he has overlooked me in collecting the fares, for the street-car company is a soulless corporation; and besides, I need the money more than it does.

12. No convicts are allowed to vote; A. is not a convict, hence he may vote.

13. No convicts are allowed to vote; A. is not allowed to vote, and so is evidently a convict,

14. In 1313 Dante was in Pisa, where Petrarch saw him as a child.

15. The theory of punishment by consequences must be correct, for it is advocated by no less a philosopher than Herbert Spencer.

16. Emmanuel Chrysoloras came on an embassy from the Byzantine emperor to solicit aid against the Turks in 1396.

17. The numerous appeals made in the name of charity would bankrupt a person who listened to them; hence it is a good rule for a person who wishes to lay by a competency to turn a deaf ear to every such appeal.

18. The verdict is wrong, for one of the jurors is not a man of good judgment.

19. We know that Deity exists because of our feelings of dependence upon a higher power; and we know that these feelings are indications of the truth because they are the expression of the divine element in our nature.

20. Some professing Christians must be inhumane; for some who professed Christianity have been persecutors, and all persecutors are inhumane.

21. Consider the record of the prisoner at the bar! Is he not a man who might well have committed the crime with which he is charged? Is he not an habitual drunkard? Was he not on one occasion arrested for non-support? Did he not at another time desert his wife and baby? Look at him well! Would any member of this intelligent jury wish his wife or daughter to be present in the same room with him? Then let a verdict be rendered in accordance with the impression that this creature, too low almost for our contempt, makes upon your refined sensibilities.

22. Every law does some injustice; injustice is morally reprehensible, therefore all laws should be repealed.

23. Space must be infinite, for it is impossible to think of a limited space that is not bounded by further space lying beyond it.

24. If we spill a peck of wheat upon the floor we hear a sound of the falling grains; hence each grain must make a sound; for otherwise how should we hear the whole peck, since it is composed only of the several grains?

25. It is the right of every one to promulgate his honest personal convictions in religious matters; hence the teacher has a right to teach her religious creed in the school-room.

26. The recent panic occurred just after the President announced his policy regarding corporations engaged in interstate commerce; therefore the President is to blame for the panic.

27. I know that I ought to make this sacrifice because I feel under obligations to make it.

28. If this act is unjust, I may trust my conscience to disapprove of it; and it must be unjust, for my conscience disapproves of it.

29. It is impossible to be a good shot without having a steady hand; John has a steady hand; he is capable, therefore, of becoming a good shot. (Minto.)

30. Mary is wise, for she has a keen sense of humor; and many wise persons are characterized by a sense of humor.

31. Lessons must arouse interest in order to be easily understood; psychology arouses interest and is therefore easily understood.

32. Persons who have good memories make ready linguists; hence B, whose memory is not good, can never hope to be a good linguist.

33. Some wise men have been philosophers; Socrates was a wise man, and therefore was a philosopher.

34. Most wise men have been also men of the highest morality; and so Solomon must have been a very moral man, for he was certainly wise.

35. If a body moves, it must move either where it is or where it is not. But a body cannot move where it is; neither can it move where it is not. Therefore motion is impossible. (Zeno's argument against motion.)

36. If the theory of evolution is harmonious with revealed religion, it contains no new truth; if it is at variance with it, it is untrue; hence it either contains no new truth or is untrue.

37. Punishment by consequences is an important means of moral improvement, and hence should be exclusively employed in the school-room.

38. Debating is a splendid method of developing practical logicians; Miss B. exhibits the skill of the practical logician in her teaching, and must accordingly have debated a great deal.

39. Interest centres upon objects that give strong sense-stimuli; hence in order to command the pupils' interest, the teacher must make a strong appeal to the senses. Logic does not make this kind of appeal, and hence it cannot arouse interest.

40. Human actions may be divided into two classes, moral and immoral; the former deserve reward, the latter punishment; hence every human action should either be rewarded or punished.

41. All colors are physical phenomena; but no sound is a color, therefore no sound is a physical phenomenon. (Creighton.)

42. Some mineral compounds are not decomposed by heat; all organic substances are decomposed by heat; therefore no organic substances are mineral compounds. (Jevons.)

43. The person who purchases some securities at the present low market price is sure to make a big profit; I have purchased some securities in the present low market; therefore I am sure to make a big profit.

44. Unpleasant experiences sometimes result to our advantage; for it is certainly an unpleasant experience to have one's teeth filled, and yet it is an advantage.

45. If pain is unnecessary, it is an evil; and if necessary, it is a curse; but it is either necessary or unnecessary, and so it is either an evil or a curse.

46. If a teacher has natural talent for her work, she can succeed without special training, and if she has no natural talent, she will not succeed with it; hence special training is unnecessary.

47. The indifferent pupil will never make a scholar. H. is no scholar, and we may naturally infer that it is due to indifference.

48. To reach their present development the civilized peoples have passed through numerous stages called "*culture-epochs.*" What we may call *race-education* having gone on in this way, some conclude that this, nature's plan, should be used in educating the individual. Hence the practice in some schools of employing a curriculum which puts the child through the chief culture-stages.

49. Some sluggards are capable of occasional spurts of energy; but sluggards are parasites upon society; hence persons who are capable of sudden spurts of energy are sometimes parasites upon society.

50. It is wrong for me to conceal the bad tidings from my sick wife because deception would wreck society if universally practised.

51. "When I look over the footlights and see your bright and interested faces I feel as if I would like to lean over and shake hands with every one." ("Curtain speech" by Joe Jefferson.)

52. "He proves, what every one will admit, that the American corporation is a good thing, that it has developed the country, that in the main it has been of advantage to all classes. In this way he tries to give the impression that what is true of the beneficial and law-abiding corporation is true of the marauding company, and that the attack on the lawbreakers is a general attack on corporations as such, and therefore a 'raid on prosperity.'" (From an editorial in a financial paper.)

53. "Quacks of every kind and name always make much of the unquestionable fact that the regular doctors often condemn to-day what they commended yesterday, and yet the essential difference between medical orthodoxy and medical heterodoxy is that the one never claims to have reached perfection, while the other always does—and in addition makes the logical leap of asserting that because one thing was wrong another thing is right." (From a newspaper editorial.)

54. "When we bear in mind that Britain has now not one peculiar mammal, and France but few distinct from those of Germany, and so with Hungary, Spain, etc., but that each of these kingdoms possesses several peculiar breeds of cattle, sheep, etc., we must admit that many domestic breeds must have originated in Europe; for whence otherwise could they have been derived?" (Darwin *Origin of Species*, Ch. I, p. 17.)

55. Admitting that you prove a body impenetrable if you can find no force that penetrates it, may it not be argued that you prove an ideal undesirable when you find nobody desiring it?

56. *Touchstone.* Why, if thou never wast at court, thou never sawest good manners; if thou never sawest good manners, then thy manners must be wicked; and wickedness is sin, and sin is damnation. Thou art in a parlous state, shepherd. (*As You Like It*, III, ii.)

57. The pedagogue was an attendant of an Athenian boy who escorted him to and from school and saw that he behaved himself. (From a recent examination paper.)

58. The act is morally wrong because it is opposed to sound ethical principles.

PART V.—INDUCTIVE INFERENCE

CHAPTER XV.—GENERAL NATURE AND FUNCTION OF INDUCTION

88. Importance and Difficulty of Induction.—In the general discussion of inference in chapter IX, the great importance of induction was seen to be due to its use in organizing experience. All knowledge-forms that grow out of repeated contact with our environment, whether they are concepts, propositions, "thoughts," principles, maxims, laws, or other modes of generalization and abstraction—in fact, everything that is neither motor nor emotional—are the direct products of inductive activities. It is customary, however, to limit the term in logic to conscious and careful processes of organizing our experience. From this point of view induction is the sponsor for all exact knowledge, for without it there would be no store of conceptual experience, and the mind would be limited to perceiving only what occupies the present moment of consciousness.

Notwithstanding that induction is of primary importance and seems to condition deduction, it is more difficult to understand than the latter. We have been practising it from earliest childhood; yet it requires the greatest intellectual skill to arrive at scientifically correct inductions. Two stages of induction may be dis-

tinguished: first, *informal induction, where generalizations are made without any special consciousness of the aim or purpose in view;* and second, *formal or scientific induction, where there is some special problem to work out or some special generalization to be made.* The former characterizes the inductive activities of childhood and also of maturity, whenever no special aim in the organization of knowledge is before the mind. The latter is the careful induction in use in discovery. It gives us scientific as distinct from common-sense knowledge. Children, for the most part, organize their experiences without any clear notion of method or aim. What is not given to them already organized by their older companions and mentors is accumulated in a naïve and unsystematic fashion. The business of childhood is to establish the common basis of practical knowledge that is essential to life. Mature people, having acquired this basis of knowledge during childhood, can devote their attention to the more conscious and less instinctive ends of scientific inquiry. They can make themselves expert in some field of knowledge, advancing beyond what has hitherto been accomplished. Having experience of the frequent failures of naïve induction to solve its problems, they can devise methods of inductive inquiry that are especially adapted to the peculiar problems they are investigating. It is this latter form of induction that gives the serious trouble. For while informal induction sets to work with no conscious problem, and so does not feel the need of a special mode of operation, formal induction, with its end clearly in view, is constantly fearful of the insufficiency of the means.

Then also the difficulty is increased by reason of the fact that every inductive problem requires a more or less special method. The data are complex and confused. The plan that works with one set of data will be likely to fail with another. Every instance examined in an inductive inquiry departs a little from every other, yet is, in many respects, similar to the others. The utmost care must be bestowed upon each particular. To follow such an investigation as the scientist undertakes, with the hope of learning his method, is really a species of apprenticeship. All that logic can hope to do in view of these difficulties is to indicate the nature and forms of induction, point out the most generally applicable methods, and warn the student against the more common inductive fallacies which are liable to vitiate a formal process. The logician cannot pretend to make an accomplished scientist by these means. This is the problem of the teacher of the science. He can, however, make any student conscious of the methods by which the most significant triumphs of science have been won; and he can emphasize the danger that lurks in the inductive fallacies.

89. Kinds of Induction.—The most common forms of induction are *perfect enumerative induction, imperfect enumerative induction,* and *methods of determining causal relation between events.* Perfect enumerative induction gives us absolutely sure knowledge. It occurs *wherever the problem is so limited in its nature or by artificial conditions that the generalization reached in the conclusion or statement of principle applies to no more instances than have been examined.* For example,

knowing that all the planets of our system have been observed, we may state the general proposition that *all the planets of our solar system move around the sun in elliptical orbits.* It may be objected to this kind of induction that it violates our definition of induction because the proposition which states the truth under consideration refers to no more cases than were examined as a preliminary to formulating it; hence that there has not been the required increase in generality. This objection is merely a formal one and is not really valid; for though the truth is applicable to no instances beyond those enumerated in discovering it, yet the inductive activity lies in just the fact that those instances ceased to be considered as several separate items at the conclusion of the investigation, and became merged in one general truth.

Imperfect enumerative induction occurs whenever, from an examination of a certain part of all instances, we reach a conclusion or statement of principle that we regard as applying also to similar instances not yet examined. Imperfect induction does not give us perfect knowledge, because the instances not yet examined may vary so much from the norm that the generalization will not apply to them. The skill in using imperfect induction lies in the ability to use the generalization only upon new cases to which it applies, or, in other words, to detect in each new instance that seems like those which have yielded the generalization, whether the seeming similarity is profound or only superficial.

The methods of determining causal relation aim to prove uniformities in the operations of nature, that is, to

prove what is an indispensable condition (the cause) of a given occurrence, or what is an inevitable consequence (the effect) of a given occurrence. They have been used from time immemorial, and their novelty consists only in the formulation which was given them by John Stuart Mill, the English philosopher, whose name is often attached to them (*Mill's Methods* or *Canons*).

90. METHODS OF INDUCTIVE PROCEDURE.—Before beginning the detailed study of the above forms of induction, it is necessary to speak of the general principles of inductive method, and to see how methodical investigation differs from our ordinary hit-and-miss manner of acquiring experience.

The first form of scientific method to be mentioned is *scientific observation, by which is meant simply directed and sustained attention employed with a view of getting exact knowledge, usually, though not necessarily, of objective facts.* Ordinary observation does not differ greatly from scientific observation. But in ordinary observation the attention flits from one thing to another at the behest of our temporary interests. In scientific observation the mind is under the guidance of some end to be attained by the investigation; hence there is permanent interest and greater control.

In using scientific observation one may either await the presentation of the facts in their natural order and setting, or he may in a measure control the order of nature and isolate the facts from their customary setting. In the latter case we have *experiment* accompanying observation. *Experiment is scientific observation directed to phenomena that are artificially controlled as to*

order of occurrence, surrounding conditions (*setting*), *or both.* The following table makes this relation between observation alone and observation with experiment clear:

Scientific observation	Following the natural order and setting.
	Modifying the natural order and setting, *i. e.*, experiment.

The great advantage of experiment is that thereby the phenomenon under observation is hastened, and that it may be isolated from complicated and confused settings. But it is by no means always possible or even advisable to use experiment; for in some cases it cannot be used, and in others it changes the phenomenon under observation. As science advances, however, increased skill in technique and invention causes experiment to encroach upon mere observation. Indeed, the state of advance reached by any science which deals with material facts may be pretty well measured by the extent to which experiment is employed.

Experiment is greatly aided by *instrumentation, by which is meant the application of mechanical appliances to the fact under investigation.* Scientific instruments are designed in the main either to aid the senses or to analyze the object under investigation. Since the eye is by far the most important sense-organ in observation, the instruments that have yielded the most benefit are the microscope, which magnifies the minute until it is brought within the range of vision, and the telescope, which magnifies the very distant until the eye sees it as though nearer. There are also numerous types of instruments for magnifying and recording infinitesimal

movements. These have been used to great advantage in studying biological facts and the physiological bearings of mental states (experimental psychology). Among instruments used in analysis and dissecting are the various equipment of the chemical, physical, and biological laboratories. Instruments may modify the phenomenon only for the observer; or they may change its nature somewhat; or they may do both. The first occurs when a telescope is used; the second, when a tissue is dissected from its natural place in the body and undergoes slight chemical change.

A phenomenon (plural *phenomena*) *is any fact or event which excites scientific curiosity, secures scientific observation, and challenges explanation.*

Scientific technique is the skilled procedure employed by the expert in any field of investigation where material phenomena are under observation.

The scientist must not only be a master of method of observation, but he must also keep exact records of all the facts which come to his notice. And since it is necessary to communicate his research to his colleagues in scientific work, he must also know how to tabulate his results. The purpose of recording is to keep the phenomena investigated for future reference, so as to compare them with like cases that may come with further research. The purpose of tabulation is to show the truth that the investigation has brought out, together with a brief review of the instances upon which it is based, so that co-workers in the same field may test the validity of the generalization. Almost every science has its own special method of record and tabulation; but

any good method must state the general truth and its grounds in clear and technical language and in logical arrangement.

91. Guiding Lights of Investigation.—No mere crowding of facts before the attention suffices to start or to guide scientific research. The facts must call forth an immediate response from the mind in the way of an excited and active imagination. As Professor Tyndall says in his essay on the "Scientific Use of the Imagination," it is the appeal which the facts of observation make to a fertile and suggestible mind, rather than the facts as such, that counts in science. Millions of apples had fallen before the eyes of men before Sir Isaac Newton brought to his celebrated falling apple a mind responsive to this familiar fact—a mind whose imagination was kindled by a trivial experience that had been to those going before him as was the primrose to Peter Bell, of whom Wordsworth tells in the lines:

> A primrose by a river's brim
> A yellow primrose was to him,
> And it was nothing more.

The place of imagination is eloquently set forth in the words of Tyndall:

> *There is in the human intellect a power of expansion—I might almost call it a power of creation—which is brought into play by the simple brooding upon facts. The legend of the Spirit brooding over chaos may have originated in a knowledge of this power.*[1]

[1] *Fragments of Science*, N. Y., 1871, p. 132.

And again:

Bounded and conditioned by co-operant reason, imagination becomes the mightiest instrument of the physical discoverer. Newton's passage from a falling apple to a falling moon was, at the outset, a leap of the imagination.[1]

The imagination here referred to is not vague fancy, but the power of combination residing in the intellect when it is dominated by a distinct purpose of rendering the most efficient aid in the advancement of human knowledge and of the dominion of man over nature. It neither goes far ahead of nor trails far behind the facts; rather it proceeds with those facts. It does not read into the facts its preconceptions, but is ever ready to abandon its dearest anticipations if the facts fail to support them. It is as patient as the slow processes of scientific observation require, yet it is alert to follow the slightest suggestion given by observation and experiment. It is of equal value with observation and with reason. Indeed, we may call these three, Observation, Reason, and Imagination, the guiding lights to all fruitful scientific inquiry. One cannot prove productive working without the others.

The earlier post-Baconian conception that all induction must move slowly and painfully on from fact to fact, with no interpolation of "guesswork," is utterly rejected by the ablest modern scientists; Huxley, an illustrious name in the annals of science, maintaining:

It is a favorite popular delusion that the scientific inquirer is under a sort of moral obligation to abstain from going beyond that

[1] *Ibid.*, p. 130.

generalization of observed facts which is absurdly called "Baconian" induction. But any one who is practically acquainted with scientific work is aware that those who refuse to go beyond fact rarely get as far as fact; and any one who has studied the history of science knows that almost every great step therein has been made by "anticipation of nature," that is, by the invention of hypotheses which, though verifiable, often had little foundation to start with, and not infrequently, in spite of a long career of usefulness, turned out to be wholly erroneous in the long run.[1]

92. HYPOTHESIS AND THEORY.—The imagination is especially requisite in the formation of what scientists call "working hypotheses," and also in the development of these into scientific theory. *An hypothesis is a supposition in explanation of facts, whose cause or reason is not yet definitely and certainly known. A theory is simply an hypothesis rendered more probable by the fact that continued investigation favors it.* The terms *theory* and *hypothesis* are often used interchangeably, even by scientific men, though the most careful writers observe the distinction above made. Thus it is that careful writers still speak of the *Nebular Hypothesis* because not enough facts in support of it are directly observable to render it more than a plausible explanation as to what might conceivably be the mode of formation of our solar system. It is also more proper to allude to the notion that Mars is inhabited as an hypothesis; for only a few directly observable facts lend themselves to its support. On the other hand, it is permissible and preferable to speak of the *Theory of Evolution,* because an immense number of observations from widely different but related fields of investigation offer themselves in

[1] Huxley, *Methods and Results,* N. Y., 1896, p. 62.

direct support of the conception. And again we may with propriety speak of the *Theory that tides are caused by lunar attraction,* for the vast number of instances in which tides have been found to vary with the moon makes this explanation a practical certainty. But at best an hypothesis shades off insensibly into a theory; for it is practically impossible to say just how much evidence must accumulate in favor of an hypothesis before it advances to the dignity of a theory. One thing is sure though: an hypothesis must "go on working," with little evidence making against it, and with a tendency to offer a plausible explanation of related groups of fact, in order to give it the high degree of probability to which a scientific theory lays claim. Referring again to the Theory of Evolution, it is fully as much owing to its applicability to a wide range of categories of fact in which development could be detected, as to the mere heaping up of direct evidence within a narrow field, that it has finally come to be accepted as a theory. It explains the facts of physical development, both in plant and animal life, as no other conception has done; it explains the facts of social, economic, and historical development; it explains the formation of our globe in the geological past; it explains mental development both in the individual and the race. With all this to its credit, scientists have at length accepted it in spite of numerous "missing links" in the chain of evidence, and despite its repugnance to the ethical and religious conceptions that it has more or less displaced.

In all of this trial explanation we have the function of imagination, which suggests the working hypothesis in

tentative explanation of a group of facts and as a beacon light to further investigation. As long as continued research seems favorable to it, it secures more general assent and passes over into theory. The following graphic description of the development of a working hypothesis is given by Professor Tyndall:

> *If in all the multiplied varieties of these phenomena* (the phenomena of light), *including those of the most remote and entangled description, this fundamental conception* (the hypothesis that there exists a medium of great elasticity and small density, called *ether*) *always brings us face to face with the truth; if no contradictions to our deductions from it be found in external Nature, but on all sides agreement and verification; if, moreover, . . . it has actually forced upon our attention phenomena which no eye had previously seen, and which no mind had previously imagined, such a conception, which never disappoints us, but always lands us on the solid shores of fact, must, we think, be something more than a mere figment of the scientific fancy. In forming it that composite and creative unity in which reason and imagination are together blent has, we believe, led us into a world not less real than that of the senses, and of which the world of sense itself is the suggestion and justification.*[1]

All great scientists excel in "that composite and creative unity of reason and imagination," as is shown by the following account of Darwin's method of work, given us by his son:

> *He often said that no one could be a good observer unless he was an active theorizer. It was as though he were charged with theorizing power ready to flow into any channel on the slightest disturbance, so that no fact, however small, could avoid releasing a stream of theory, and thus the fact became magnified into impor-*

[1] *Loc. cit.*, p. 133.

tance. In this way it naturally happened that many untenable theories occurred to him; but fortunately his richness of imagination was equalled by his power of judging and condemning the thoughts that occurred to him. He was just to his theories, and did not condemn them unheard; and so it happened that he was willing to test what would seem to most people not at all worth testing. These rather wild trials he called "fool's experiments," and enjoyed extremely. As an example, I may mention that, finding the cotyledons of Biophytum to be highly sensitive to vibrations of the table, he fancied that they might perceive the vibrations of sound, and therefore made me play my bassoon close to a plant. The love of experiment was very strong in him, and I can remember the way he would say, "I sha'n't be easy till I have tried it," as if an outside force were driving him.[1]

In this description of the experimental method of the nineteenth century's greatest scientist is to be seen a wonderful imagination working under the guidance of definite purpose and the control of reason. Here is no mere idle fancy, wandering hither and yon at the dictates of temporary interest, but a power of imagination that strives toward a worthy goal. It is such imagination that must be depended upon for the framing of useful hypotheses.

An hypothesis renders two services to the investigator: first, it guides experiment by selecting from among the antecedents of a phenomenon those that seem most relevant to it, and by confining investigation along that line; second, it furnishes a tentative explanation of facts whose antecedents are so obscure or complex that they cannot be certainly detected or properly isolated for experimental treatment. As a consequence of this last

[1] *Life and Letters of Charles Darwin,* Vol. I, p. 126.

use, it follows that we have to depend more and more upon hypothesis the wider the range of facts that we attempt to explain together on one principle. Thus it happens that philosophical explanations, which seek for first principles that will harmonize the widest ranges of fact, are almost entirely hypothetical.

There is one point about the use of an hypothesis that should be observed. Every hypothesis should develop into a theory as facts in its support accumulate; or, if facts fail, it should be rejected. It must be borne in mind that an hypothesis is in no sense final; it is a mere preliminary to a better understanding of something obscure. Unless future research along the line of investigation which prompted its framing tends to give it support, it may be regarded as of doubtful worth. Now the mind cannot indulge in entertaining mere playthings of the imagination without the danger of mistaking them for realities. An hypothesis which wants support in facts may make a strong appeal to our feelings. If we indulge in its contemplation time and time again we grow to believe it because we want to believe it. And besides, the tendency to a habit of thought respecting it results in ultra-conservatism and an unwillingness to consider more plausible explanations. It is this tendency to maintain an hypothesis beyond its period of usefulness that accounts for the slow growth of the scientific spirit in the Middle Ages. Notions that chimed in with the religious beliefs of those days were adhered to with a fidelity that deserved a better object. Hence, conceptions like the Ptolemaic hypothesis of the sidereal movements and the mediæval hypothesis as to the shape of

the earth were exceedingly slow to give way to the correct interpretations of Copernicus and others. It is really a fallacy to hold to an idea after it has grown decrepit with a futile struggle to root itself among our permanent truths. We should be logical enough to give it up in favor of something newer and better.

93. Requirements of a Good Hypothesis.—No mere rough and fanciful guess will meet the requirements which scientists lay down for an hypothesis. To be worthy of consideration it must conform more or less accurately to the following conditions, most of which are really demands of logic as well as of science:

1. *It must be simple.*
2. *It must be plausible.*
3. *It must be adapted to the facts requiring explanation.*
4. *It must be general enough to cover all the facts requiring explanation.*
5. *It must be self-consistent from a logical stand-point.*
6. *It must be capable of being proved or disproved.*
7. *It must harmonize with accepted explanations in related fields of research.*

A moment's consideration of these suggestions and we shall dismiss this topic. *Simplicity* is requisite in a scientific hypothesis because it aims to make the obscure facts of observation clearer. The complex is the obscure and the simple is the clear. *Plausibility* means believableness. No trial explanation is worth considering if it calls upon too great a spirit of credulity. True it is that many absurd hypotheses are entertained by large groups of people, but they are not *scientific* hypotheses. *Adaptableness to the facts* is necessary because any

hypothesis, be it ever so attractive, is of no value unless it can be shown to have a direct bearing upon some definite group of facts. *Generality sufficient to cover the facts* is indispensable because the business of the hypothesis is to furnish a tentative explanation of *all* the facts, not merely part of them. *Internal consistency* is essential because the aim being to assist the reason and enlighten the intellect no hypothesis does this which harbors logical inconsistencies. *An hypothesis must be capable of proof or disproof* because it is constructed with an aim of ultimate knowledge and truth; if a true explanation, we must one day know it; if not, we must have definite grounds for abandoning it in favor of something else. *Harmony with explanations in related fields of research* is necessary because an hypothesis, being at best only a guess, should not undermine better established conceptions that time has shown to be essentially true. We must not press this last point too far, however; for it has often happened in the history of science that the most firmly rooted beliefs have had to give way before a more reasonable hypothesis which was irreconcilable with them, and thus science has progressed and been the gainer.

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REVIEW QUESTIONS

1. *Define induction.*
2. *What is the general purpose of induction?*
3. *Why is induction important?*
4. *What two stages of induction may be recognized and what are their distinguishing characteristics?*
5. *Why is induction difficult?*
6. *What are the kinds of induction with which logic is chiefly concerned?*
7. *Define perfect enumerative induction.*
8. *Define imperfect enumerative induction.*
9. *What is the aim of the methods of proof of causal relation?*
10. *Define scientific observation and distinguish it from ordinary observation.*
11. *Define experiment.*
12. *Discuss the advantage and the limitations of experiment.*
13. *How may we gauge the advancement of a science?*
14. *Define instrumentation, and discuss the chief aid rendered by scientific instruments.*
15. *Define the term phenomenon.*
16. *Define the term scientific technique.*
17. *Why are recording and tabulation of importance in science?*
18. *What kind of imagination is necessary in science and how is it to be used?*
19. *Define the term hypothesis.*
20. *Define the term theory.*
21. *How would you distinguish between the two?*
22. *Why should we now speak of the Theory rather than the Hypothesis of Evolution?*
23. *What power of the mind frames hypotheses?*
24. *What does Professor Tyndall say about this power?*
25. *How was Darwin accustomed to theorize?*
26. *What is the value of an hypothesis to the scientist?*
27. *What should an hypothesis do if we are to entertain it?*
28. *State and discuss the requirements of a good hypothesis.*

CHAPTER XVI.—COMMON FORMS OF INDUCTION: ENUMERATION, STATISTICS, PROBABILITY, ANALOGY

94. Perfect Enumerative Induction.—It has been made clear in the previous chapter that in perfect induction the generalization expressed as a conclusion applies to no instances beyond those enumerated and examined in forming it. Perfect induction is perfect knowledge, and is hence the kind that would be most satisfactory in regard to everything, but, unfortunately, human experience is far too narrow to permit our arriving at this goal except in unusually favorable circumstances. Perfect induction in natural science can result only when the cases studied are limited in number and when no new cases will be produced. As this does not often happen we can scarcely ever attain absolute certainty in regard to natural phenomena.

We are here speaking of *enumerative* induction. The word enumeration may possibly be a little misleading to literal-minded students of logic who at once think of the precision aimed at in arithmetical enumeration. *In logic enumeration does not imply exact counting; it means rather the mental tabulation or noting of instances.* Hence, when enumeration or enumerative induction are mentioned here they refer to the mental habit of taking note of the characteristics in like experiences with the idea of estimating roughly what proportion of the instances show the same characteristics.

95. Imperfect Enumerative Induction.—It has been shown that imperfect induction examines a certain

number of particulars, finds the common characteristics, and generalizes a principle which applies to like cases beyond those examined in forming it. Its advantage over perfect induction lies in its wider range of applicability. Were our inductive generalizations limited to complete experience of the instances which it is meant the generalization shall cover, our fund of laws and principles would be vastly less than is actually the case. *An act of imperfect enumerative induction is performed whenever, as the result of having investigated certain more or less typical cases, a conclusion is asserted that applies to similar cases not yet investigated.* This kind of induction is particularly useful in giving uniform qualities, modes of behavior, functions, and other descriptive uniformities that suggest rather than discover causes.

Its method is as follows: A somewhat rough estimate of the number of like cases is made, and the proportion of such cases to all possible instances is noted. Should all possible cases have been included in the examination and no exceptions found, we are entitled to perfect enumerative induction covering all the kind to which the generalization may, under any circumstances, apply. In other words, we have perfect knowledge. Should some exceptions appear in that part of the entire field actually examined in attempting to reach a generalization, we have no true form of induction, but only a set of data to be treated by the Theory of Probability. But if no exceptions occur within the facts observed, while at the same time we know that our examination has included only part of all facts of that kind, we have

an opportunity to make an imperfect inductive generalization. Then it would be founded upon the facts actually examined but would be made to apply to all instances of that kind, even without examining them all. In this case our knowledge is not perfect, because among the cases not examined but to which the generalization is applied there may be exceptions. Skill in using imperfect induction lies in the ability of the investigator to pick out typical or representative instances for examination, and to avoid applying the generalization to atypical instances which really present so many and important departures from the typical cases that they should be classified as another class.

96. The Inductive "Leap."—The need of economizing effort soon brings the scientific investigator to a place where he must stop examining new cases. At this stage one of two courses lies before him. It has been shown that he may formulate a generalization covering only the cases investigated and reach a perfect inductive generalization. But his purpose in making the inquiry was not to find out about only the few cases he found opportunity to examine. He also wanted to know the truth respecting the numerous instances that were not subject to his direct scrutiny. Hence he usually pursues the second course. Considering the cases examined as typical, he expresses his generalization in such a way as to extend it beyond the cases under inspection to other possible cases of a similar nature. In other words, he forms an imperfect inductive generalization. Now the question arises, What warrant has he for making his generalization broader than the

instances examined? The warrant is that state of mind known as faith or belief. Having found uniformity in his experiences, the scientist expects uniformity in possible future experiences. Just at this juncture the mind turns prophet, foretelling what it *will* experience from what it *has experienced.* This is called by logicians the *inductive "leap"* or *"hazard,"* these figures of speech indicating that we here pass beyond the strictly known and prophesy what our tendency to believe in the reasonable leads us to expect.

97. THE POSTULATE OF THE UNIFORMITY OF NATURE. —The warrant for this leap of the mind from what has been to what will be depends upon a postulate or necessary assumption that what has been will be unless we can see an obvious reason for its not continuing to be. In other words: *any event in our experience may be regarded as an effect of some cause; and unless we can see why that cause has ceased to be operative we look for the effect.* While this is a postulate of our minds it is also one of our broadest inductions. All our experience seems to favor it. Nature continually reproduces much the same forms because the cause continues to work, and will continue operative until counteracted by some other cause. This postulate, strengthened by induction, really implies that our minds have a structure which assumes that experience, as it comes to us, expresses rationality in the plan of the universe—a rationality superior to but at the same time similar in kind to our own mind. Hence it may be said that the principle of the uniformity of nature is simply a somewhat popular statement of the conviction that the uni-

verse is the expression of a rational plan to a mind capable of conceiving such a plan.

98. The Syllogism in Which Imperfect Enumerative Induction May be Formally Expressed.—All imperfect induction may be expressed in the following standard logical form:

Case 1, Case 2, etc. . . . nth Case have the character in question;
Case 1, Case 2, etc. . . . nth Case are types of all cases;
Therefore all cases have the character in question.

Or, more concretely stated:

This, that, and the other house-fly have two wings;
This, that, and the other house-fly are representative of the species;
Therefore house-flies have two wings.

99. Relation between Induction and Mathematical Processes.—Arithmetical processes may be considered cases of perfect enumerative induction, for the total represented by the number is a sum of homogeneous units: as when we say forty-five sheep we count forty-five times one unit (the units being indifferent) and exhaust all cases in the count. The same is true of the use of geometrical figures in demonstrations: the theorem is proved for one figure, but that figure is like the unit in arithmetic in the fact that considerations of size are ignored, one figure being interchangeable with any other that preserves the relation of geometrical elements (lines and angles) required by the construction.

We are not to assume, however, that this exhausts the use of induction in the teaching and learning of mathematics. To acquire mathematical knowledge means

to acquire a given form of experience; and this implies the gradual accumulation in the mind of data which are compared and generalized in proper inductive form as the basis of principles. Deduction is also much used in mathematics: as when special forms of construction are considered as problems determined by broad general theorems which the student is to use to prove them.

100. Estimation of Probability.—Many events are so complex in their antecedents, and seem from casual observation to show so little regularity of recurrence under what are apparently the same conditions, that it is difficult to discover uniformity in them. Such is the case respecting many things that lie in the future. Our judgment of what the future is to bring is based upon past experience. But we know it often happens that our calculations regarding occurrences that are to come go astray. Whether I shall start upon a long journey to-morrow depends upon whether I get a telegram stating that a certain piece of business has to be transacted. I cannot be certain that the weather will stay clear to-morrow even though there be no cloud in the sky to-day. If I toss a coin I cannot tell whether it will fall heads or tails. If I invest in shares in a commercial enterprise I may make or I may lose. And so it is in countless instances. Perfect knowledge of all the antecedent circumstances would disclose the future with all its endless happenings; but, lacking such knowledge, we must gamble on the issue, as it were; we must make a rough or an exact estimate of the chances of an event's happening as over against its not happening, and be governed by this estimate. To estimate chances

upon a basis of rough or exact counting or enumeration of cases happening in a given way, as opposed to all cases, belongs to the *Theory of Probability,* and this theory is derived necessarily through inductive processes. In case the whole number of possibilities is definitely known, the chance of any event's occurrence is expressed by a fraction whose numerator represents the number of times the event in question has happened, and whose denominator represents the entire field of possibility. Thus there are fifty-two possibilities in drawing a card from a deck; meantime there are only four aces; the chance of drawing an ace is therefore $\frac{4}{52}$ or $\frac{1}{13}$. So, also, since there are six faces on a die and only one face with one spot, the chance of throwing an ace with an unloaded die is one-sixth. In tossing a coin the chance for heads is one-half. Here we have a simple calculation with the possibilities limited and known, and the ratio of the event to the possibilities representable by a simple fraction. In most cases the situation is far more complicated, and the probability of a given event cannot be so readily and definitely determined. Thus the chance of its being clear to-morrow night at first sight appears to be one-half. But complicated meteoric conditions that the average person cannot comprehend are acting as causes which may play havoc with so simple an estimate of chances. In complex cases it is necessary to observe and record, or else to estimate with considerable accuracy, a large number of cases and determine the average ratio of the event to all cases happening in that field of investigation. Suppose that I am interested in determining the average of black sheep in the flocks of a given

locality. I examine, say, twenty flocks of one hundred sheep each. They run, let us suppose, four blacks in the first flock, six in the second, seven in the third, three in the fourth, five in the fifth, four in the sixth, and so on. Adding the total number of sheep in all the flocks and dividing by twenty gives, we will say, an average of five and a half sheep per flock of one hundred. Assuming by imperfect induction that this proportion will hold through the locality under investigation, I can express the chance of black sheep in that country-side by the fraction $\frac{11}{200}$. In order to collect sufficient data to make such an investigation reliable we have to collect statistics, which are simply carefully taken numerical records of classes of occurrences that are too complicated to be understood by direct observation.

101. USE OF STATISTICS.—*Statistics depend upon the enumeration and recording of a great number of facts of a given class.* It is noticeable that the phenomena to which statistics are to be applied are too complex for direct observation or else that the direct methods of experiment and observation would destroy the facts. It is also noteworthy that the facts are countable, because they can be treated as abstract individual units. Again, the cause of the phenomena under investigation by the statistical methods does not disclose itself by ordinary scientific treatment. Vital phenomena are especially adapted to statistical treatment. The same is even more the case with the species of vital phenomena known as sociological and economic. Here the facts are extremely complicated; to experiment in order to know them better would interfere with them; they may be

treated as abstract units; and finally their causal explanations are in the nature of conjecture and hypothesis rather than settled truths. Statistics are helpful to science, first, because they group the facts so that the mind can grasp them clearly, and second, because they often reveal quantitative variations that suggest causal connections. To illustrate the points that have just been brought out, we may consider several cases where statistics are useful in giving a more accurate knowledge of the phenomena in question than could be obtained either by casual observation or by the ordinary scientific methods. The birth-rate for the different months in the year for a period of five years covers a field of facts entirely too vast for ordinary observation. The memory would be swamped in trying to retain data so numerous, even were it possible for any small group of scientists to obtain them. But by gathering records from the various localities in the United States a table of statistics may be compiled which groups the facts so that they may be dealt with by the scientist. Again, were the medical man interested in finding how many cases of hydrophobia are cured in proportion to those that result fatally, he could not resort to experiment; he must collect a record of all cases that occur in the natural course of things. Furthermore, such facts as these are amenable to abstract numerical treatment; they are units of birth, hydrophobia, etc., and not instances of the birth, recovery, or death of personalities that have special concrete interest for the investigator. Finally, the ultimate causes of variation in such phenomena cannot be readily ascertained in the ordinary way.

Why the birth-rate is higher certain months than others is not readily explainable, because the phenomenon depends in some obscure way upon a complicated causal situation whose analysis is impossible until the facts to be explained are numerically grouped. When variations coincide, time and time again we come to suspect causal connection. If a large number of records reduced to statistical form show that the birth-rate is unusually high at the same time that the cost of living is unusually low, we are led to suppose that there is some causal connection between the two phenomena. Statistics do not furnish ultimate explanation. They are only a method of putting facts into better form for explanation.

102. Reasoning by Analogy.—*Analogy is a method of reasoning whcreby we attempt to explain facts that are obscure or not subject to accurate scientific treatment by means of their apparent likeness to facts that are already explained or at least better understood.* It is essentially a method of comparing instances representing large groups of facts. One of the representative instances is pretty well understood; the other is not, but is seen to have some striking resemblance to the better explained instance; we argue that the striking resemblance implies that the explanation already admitted for the better explained instance holds also for the other. Such an argument may contemplate facts or relations between phenomena. In the early history of the attempt to explain the phenomena that held human interest this kind of explanation was resorted to almost exclusively. In fact, analogical explanations were almost the only ones until the era of modern science; and the readiness of the

mind to fall back upon them in case more satisfactory explanations cannot be had attests how strong must have been the habit of analogical argument in the past. In early times men knew the meaning of their own feelings, passions, and intellectual processes much better than the facts of nature. Hence the latter were explained after the analogy that they bore to the former. Mythology shows how the ancients peopled the earth with spirits to account for natural happenings. As experience was accumulated, recorded, and transmitted from one generation to the next, the inadequacy of these childlike interpretations of nature gradually dawned upon human consciousness. The disappointments which these mythical views of nature were fated to bring suggested the need of more careful thinking. Out of this grew science.

Though analogy can no longer be looked upon as scientific explanation, it is yet a fertile source for hypothesis, and thus it leads the way toward real explanation. Though we now look upon the guesses of the ancients as the myths of an infant race, yet they were prolific of really good explanations later on.

Analogical reasoning is well illustrated in the notion, held by certain speculative astronomers, that the planet Mars is inhabited. We know from our observations here upon the earth that certain conditions of heat, moisture, etc., produce the forms of animal and vegetable life. These conditions are due mainly to the position of the earth with respect to the sun as regards distance, inclination of axis, and times of revolution and rotation. We note that so far as we can judge at this

remote distance, the same general conditions obtain in the case of Mars. We argue that since, with these conditions, there is life upon the earth, there is also life upon Mars.

Analogical reasoning is quite liable to error owing to the readiness of the mind to seize upon conspicuous superficial similarities, and to ignore the really important details wherein the instances differ. Thus in the analogical argument in favor of the immortality of the soul, based upon the emergence of the butterfly from the chrysalis, the mind is impressed with the ugliness and inertness of the pupa and the beauty of the full-fledged insect as typical of the difference between the soul, weighed down by the imperfect body, and the soul freed from all earthly dross and dwelling in immortal bliss. But there is no notice of the fact that the marvellous change of the insect is included in a single life history and is hence a continuity of the same kind of life, whereas the change from a soul immured in a body to a soul freed from the body involves an absolutely new phenomenon, that of the disintegration and decay of the body. Another good instance of wrong analogy is seen in the following passage, quoted from Chittenden's *Physiological Economy in Nutrition:*[1]

We are too wont to compare the working-body with a machine, the boiler, engine, etc., overlooking the fact that the animal mechanism differs from the machine in at least one important respect. When we desire to set machinery in operation we must get up steam, and so a fire is started under the boiler and steam is generated in proportion as fuel is burned. The source of the energy

[1] P. 21.

made use of in moving the machinery is the extraneous combustible material introduced into the fire-box, but the energy of muscular contraction, for example, comes not from the oxidizable food material in the stomach, but from the material of the muscle itself. In other words, in the animal body it is part of the tissue framework, or material that is closely incorporated with the framework, that is burned up, and the ability to endure continued muscular strain depends upon the nutritive condition of the muscles involved, and not upon the amount of food contained in, or introduced into, the stomach.

The popular mode of argument by analogy referred to here sees that the animal body is like a steam-engine in that it is an intricate structure for converting energy. This being the likeness noticed, the argument is extended to the point of asserting that because the work of the engine bears a direct relation to the amount of fuel put under the boiler, so the working power of the body will depend upon the amount of food taken into the stomach. The error in the reasoning lies in favoring the similar function of converting energy, and ignoring the vital difference that *the body consumes its muscles to get its energy, whereas the engine does not consume its working parts but merely the coal put into the fire-box.*

Care in the use of argument from analogy implies, first, that we fully recognize that it is from instance to instance, and hence does not give us absolute knowledge but only some degree of likelihood, second, that we use our utmost efforts to determine whether any striking likeness we may detect between the two instances concerned in the argument is supported by other likenesses, and third, that we discover whether the likeness is profound or merely superficial.

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REVIEW QUESTIONS

1. *What kind of knowledge does perfect induction yield?*
2. *When may perfect induction be used in scientific inquiry?*
3. *How does logical enumeration differ from arithmetical enumeration?*
4. *What is the nature of imperfect enumerative induction?*
5. *In what ways is imperfect enumerative induction useful?*
6. *What kind of knowledge does imperfect induction give?*
7. *Wherein lies the skill in imperfect induction?*
8. *What is the inductive "leap"?*
9. *What do you understand by the postulate of the uniformity of nature, and how is it to be interpreted?*
10. *How is induction related to mathematical processes?*
11. *What do you understand by the Theory of Probability?*
12. *How do we express the probability of countable cases?*
13. *What is statistics?*
14. *When are statistics most useful? With what kind of phenomena?*
15. *Why are they helpful to science?*
16. *What is analogical reasoning?*
17. *Why is analogy still useful?*
18. *Why do we so frequently err in using analogy?*
19. *In what three ways may analogical reasoning be safeguarded?*

EXERCISES ON CHAPTER XVI

1. Name two instances of generalizations due to perfect enumerative induction.

2. Cite five cases (drawn from your own recent experience if possible) which should be generalized by imperfect induction.

3. Put each of the cases cited in answer to Exercise 2 in the form of an imperfect inductive syllogism.

4. Estimate and express by a fraction the probability of drawing an ace of diamonds from a deck of playing cards.

5. Estimate and express by a fraction the probability of making a sum of four spots by the throw of two dice from a box.

6. Estimate and express by a fraction the probability of holding five trumps in a hand of whist.

7. Assuming that you hold five trumps in whist, estimate the probability of your partner's having four trumps, and express fractionally.

8. Assuming that a friend takes a five-o'clock train four evenings out of the week, and that you have forgotten what those evenings are, what is the probability that you will meet him if you chance to take that train any particular evening?

9. Make statistics showing the blondes and brunettes in your class.

10. Make statistics showing those in your class that you would estimate to be over and under and just five feet five inches tall.

11. Make statistics showing the ages of any five members (named as Miss A, Miss B, Mr. C, etc.) of your class and find the average age.

12. Name five cases where you think that statistics could be used to advantage.

13. Cite two cases where you would employ analogical reasoning, and work out or explain the analogy.

14. In the two cases above cited (Exercise 13) indicate the degree of likelihood, try to find other similarities which support the one upon which the reasoning was based, and show whether the likeness is profound or suoerficial and why.

CHAPTER XVII.—METHODS OF DISCOVERING CAUSAL RELATION

103. Events and Experience.—What we ordinarily call by the general name of events or happenings are really interdependent factors in a net-work of experience. The untutored mind tends to see facts or events in isolation. The moon rises. The dew falls. The winds blow. The water flows in streams, moves in ocean currents, and falls in rains. Yet, wanting some sort of explanation of these natural events, the savage attributes them to the caprice of a will indefinitely like that known in himself. In other words, he relies upon a simple analogical interpretation of these changes in nature. The cultured and scientific mind is past the stage where these simple attempts at accounting for natural occurrences can satisfy. One may *believe* that such occurrences are indirectly attributable to a mind something like his own, only vastly more capable, just as the events themselves are infinitely more stupendous than his own puny undertakings. But he *knows*, as the result of numerous recorded experiences, that this presumed intelligence to which all nature is due works only indirectly, and that he must look to the connections between the events themselves for scientific explanation. He knows that the cause of a happening in the natural world is always some other happening in the same

natural world, and that therefore scientific explanation must limit itself to fathoming these causal connections, leaving the question of an *Ultimate Cause* to be solved by religious faith or philosophy.

The scientist also knows, from past experience in dealing with natural phenomena, that each event has as its immediate forerunner an almost infinite number of other events; hence, that it is his business to disentangle from this intricate complex of events that one which is the sole and indispensable condition of the happening of the event under investigation. The group of circumstances that precede an occurrence and contain among them the factor without which it would not happen is called the *antecedent* of the phenomenon. *That sole factor in the antecedent which is indispensable to the occurrence of the event under investigation is called the cause of the phenomenon.* The group of circumstances that follows an event and contains the factor which invariably follows upon the occurrence of the event under investigation is called the *consequent* of the phenomenon. *That sole factor in the consequent which invariably follows the occurrence of the event under investigation is called the effect of the phenomenon.* The term *event means any occurrence or happening.* It is almost synonymous with *phenomenon* (defined on page 202). The antecedent always contains the cause; the consequent the effect. Every cause is in its turn the effect of some other cause, and every effect is likewise the cause of some other effect; in other words, the causal series is infinite until our thinking brings us to the *First Cause*, or the Self-caused (*causa sui*). But while this is log-

ically and metaphysically true, practical science deals always with what we may call *causal couples, i. e.*, a causal pair limited to a cause and its effect. Hence, when science attempts to disclose the nature of a thing or to explain, it really does nothing more than to assign the indispensable condition of a thing or its inevitable result (its material cause or effect).

104. General Nature of the Mill Methods.—Now the *methods of determining causal connection from the stand-point of scientific explanation,* more popularly known as Mill's Methods or Canons, are really methodical procedures for analyzing the antecedent to find the causal factor or the consequent to find the effect. Their place in inductive inquiry belongs to them because from a certain number of instances examined we conclude by imperfect induction to all like cases. They prove causal connection not on a rational but on an empirical basis. In other words, they do not assign a reason why a given cause produces a given effect based upon the essential nature of the cause and effect, but merely show that a given cause has uniformly produeed a given effect, and assume by imperfect induction, utilizing the postulate of the uniformity of nature, that it will continue to produce just that effect and no other. Hence Mill's Methods are methods rather of scientific analysis than of philosophical explanation. But since they involve method, rely upon induction, and make a claim to give us practical truth they appropriately find a place in logic.

The mind having once discovered generalizations respecting cases examined referring to common qualities or functions of the

things investigated naturally supposes that there is some unity underlying them and serving as the ground of their likeness of quality or action. This unity is interpreted by science as that which uniformly conditions the things in question, or as a cause. We have here to consider the ways of analyzing the complex of events which constitutes our experience antecedent to an occurrence, and of discovering the uniformly present causal unity. The methods of distinguishing the cause also serve to determine the effect.

John Stuart Mill, the English logician, differentiated and formulated five of these methods. The rules governing them are stated in formulas to which he gave the name of canons. Though the methods are five in number, Mill himself indicates in the following passage that only two principles are involved. He says:

The simplest and most obvious modes of singling out from among the circumstances which precede or follow a phenomenon (any event), *those with which it is really connected by an invariable law, are two in number. One is by comparing together different instances in which the phenomenon occurs. The other is by comparing instances in which the phenomenon does occur, with instances in other respects similar in which it does not. These two methods may be respectively denominated the Method of Agreement and the Method of Difference.*[1]

The Joint Method of Agreement and Difference is simply a combination of the above two, while the Method of Residues and the Method of Concomitant Variations are merely modified applications of them.

105. Method of Agreement.—The Method of Agreement is used to analyze the antecedents of an event and find the only factor that is always present when that event occurs. It gives us the probable cause.

[1] *System of Logic,* Bk. III, Ch. VIII, § 1.

If we could make certain by this method alone that we had detected the only factor among the antecedents that is always present, we should have absolute scientific proof that the cause was found. Theoretically we can do this; but when it comes to actual practice scientific problems do not usually submit to such simple treatment. The antecedents are not so readily analyzed as we could hope. Hence it happens in most cases that the factors cannot be isolated completely enough to prevent the possibility that more than one is present. And it not infrequently happens that when the investigator believes he has the one factor isolated from the others, there remains involved with it a hidden factor that has entirely eluded his observation. Hence it is usual to supplement the Method of Agreement by the Method of Difference, which subtracts the factor that is suspected of having causal efficiency. If, that factor having been removed and the other antecedents being operative, the effect does not occur, we have the strongest reason to conclude that the factor which alone is present when the effect occurs and alone is absent when the effect does not occur is the cause. But even when an investigation has been carefully conducted in the way described, it must be remembered that only one instance of causal connection has been demonstrated. It is, therefore, usual among careful scientists to have a repetition of the causal demonstration by other scientists. If the results agree, there are several investigations which serve as instances for imperfect induction. By generalizing them we discover and state a law of causation. Such a law is, it must be remembered,

founded upon limited investigations, and is hence called *empirical,* which means that it is based upon limited experience, but experience that has so far proved uniform; and hence such a law has the validity of any other formula derived by imperfect induction, and no more.

Mill states the formula for the Method of Agreement in the following form, to which it is customary to give the brief name of the First Canon: "*If two or more instances of the phenomenon under investigation have only one circumstance* (factor) *in common, the circumstance in which alone all the instances agree is the cause* (*or effect*) *of the given phenomenon.*"

Jevons states the part of the formula which refers to determining the cause more briefly thus: *The sole invariable antecedent* (factor among the antecedents) *of a phenomenon is probably its cause.*" To this Creighton adds a similarly worded rule to govern the proof of the effect: "*The sole invariable consequent* (factor among the consequents) *of a phenomenon is probably its effect.*"

It should be observed that whenever throughout this discussion we speak of discovering the cause the same principle holds for discovering the effect, the language being changed to suit.

The Method of Agreement is untrustworthy under the following three conditions: First, if the phenomena which we suspect to be causally related are co-existent rather than successive. Thus it is difficult to determine by this method whether ignorance causes immorality or immorality ignorance. The two conditions appear together in numerous instances. But they co-exist. Hence many say that the one is the cause, the other the

effect, while others maintain just as stoutly that the reverse order is the true one. The probable explanation is that they interact, one of them being now the cause and again the effect. But whatever the explanation, it must be clear that in so complicated a situation the Method of Agreement would fail to disclose the cause or the effect. The second case of failure is when more than one cause may produce a given effect ("plurality of causes"), or more than one effect may ensue from a given causal factor. As an example of this we may cite the case of a pupil's correct deportment in the schoolroom, which may be due either to a temporary spasm of reform, to fear of punishment, or to hope of reward. As an example of plurality of effects we may take the case of the administration of a given quantity of chloroform, which in one instance produces nothing more serious than unconsciousness with relief from pain, while in another it results in heart failure and death. The third situation where the method fails to give satisfactory results is when either cause or effect is a group of factors rather than a single factor ("joint cause" or "joint effect"). Thus failure to pass an examination might conceivably be due to indifference, lack of time for preparation, and ill-health, all conspiring to defeat the pupil's progress. In fact, the Method of Agreement alone is most applicable when the circumstances attending the occurrence of an event are easily analyzable into independent factors, and when these factors readily present themselves in different combinations. Since this favorable situation depends rather upon the way in which nature presents the data than upon

artificial manipulation of the data, the Method of Agreement is mainly a systematic plan of observation and record rather than of experiment.

106. Example of the Method of Agreement.—Let us suppose that the symptoms of ptomaine poisoning develop among the members of three families in a given neighborhood. The attending physicians exchange experience, become interested in finding to what it is due, and start an investigation. It develops that the family of Mr. A had for supper on the evening preceding the development of the disease bread from M's bakery, butter of the J brand, home-made jelly, and oysters from X's fish market. It is found that the family of Mr. B ate on the same evening rolls from N's bakery, butter of the K brand, steak from P's market, apple-sauce, cake from N's bakery, and oysters from X's fish market. Mr. C's family on that evening had a supper composed of O's bread, L brand of butter, oysters from X's market, peaches canned at home, and chipped beef of the Y brand. Now it will be noted that in these several cases of poisoning the sole invariable factor among the antecedents of the effect (the viands eaten) is the oysters from X's market. Hence the oysters are regarded as the cause of the ptomaine poisoning.

The following case serves to illustrate this method used to determine the effect. A physician has a fever patient. He hears of a newly discovered drug that he thinks might have some effect in bringing about a cure. He administers it on three successive days and notes the changes that ensue in the patient's con-

dition. On the first day the clinical thermometer indicates a considerable lowering of the temperature, the nurse reports that the patient had a quiet sleep, and the pulse is stronger. On the second day the patient is slightly delirious, has a feeble pulse, and the thermometer shows a lowered temperature. The third day there are troublesome pains in the head, a difficulty in breathing, a brighter mental condition, and a lowered temperature. The physician argues from these observations, made let us say two hours after the administration of the medicine in each instance, that the treatment tends to allay the fever. In this case the sole invariable factor among the consequents that experience would lead one to suppose might have some connection with the medicine is the lowered temperature.

That the Method of Agreement is usually merely a step in the direction of analysis of complex circumstances so as to eliminate the irrelevant or variable factors and focus attention upon the relevant or invariable ones with a view to further investigation by other inductive methods is shown by Minto in his discussion of Wells's investigation into the cause of dew.

Comparing the numerous instances of dew appearing without visible fall of moisture, Wells found that they all agreed in the comparative coldness of the surface dewed. This was all the agreement that he established by observation; he did not carry observation to the point of determining that there was absolutely no other common circumstance: when he had simply discovered or detected that this circumstance was common to dewed surfaces, he tried next to show by reasoning from other known facts how the coldness of the surface affected the aqueous vapor of the neighboring air. He did not

establish his Theory of Dew by the Method of Agreement: but the observation of an agreement or common feature in a number of instances was a stage in the proccss by which he reached his theory.[1]

The Method of Agreement is usually simply methodical observation directed to the detection of uniformity. It suggests a theory as to the causal factor and then gives place to the more scientific methods, such as Difference and Joint Method.

A general formula for a problem in the analysis of the uniform factor from the varying antecedents may be expressed by the following letters, it being understood that they are chosen arbitrarily except that C stands for cause and E for effect. The index figure signifies the number of the instance under observation.

EFFECT KNOWN; CAUSAL FACTOR TO BE FOUND

ANTECEDENTS	CONSEQUENT
A B D F	E^1
A G H I	E^2
A J K L	E^3

CAUSE KNOWN; EFFECT TO BE FOUND

ANTECEDENT	CONSEQUENTS
C^1	A B D F
C^2	A G H I
C^3	A J K L

When the antecedents have been analyzed, the sole uniform factor A is found to be the cause of the effect under observation E. When the consequents have been analyzed, the sole uniform factor A is found to be the effect produced by the cause under observation C.

[1] Minto, *Logic*, p. 325.

The student needs to be cautioned in regard to two or three errors that he may easily make in the use of this method. First, he is quite likely to introduce factors among the antecedents or consequents that are entirely irrelevant to the case. He will avoid this tendency if he bears in mind the fact that we always approach a problem of this kind with some experience, and hence we should narrow down the group of factors investigated to those which the imagination could connect with the case. Second, the event, be it cause or effect, from which he starts has always been analyzed from the context of experience and stands isolated before the attention. His problem lies, therefore, in disentangling from a group of antecedent factors (if he is trying to determine the cause) or from a group of consequent factors (if he is trying to determine the effect) *the sole uniformly present factor.* Hence in an example illustrating the inquiry into the cause the *antecedents must be complicated,* while in an example illustrating search for the effect *the consequents must be complicated.* Third, it is much easier to find examples illustrating this method applied to determining the cause than to determining the effect. This may be owing to an almost universal habit of starting from the effect and pursuing the investigation backward to the cause. But it would also seem to indicate that the method is better adapted to determining the cause from a known effect than the effect from a known cause.

107. Method of Difference.—The second method cited by Mill is one that is peculiarly well adapted to experiment. When observation has led one to suspect that a certain factor among the antecedents of an event is its cause, it is often possible to subtract that factor by artificial means. If when the factor is absent the event under observation does not take place, we have the strongest kind of evidence that the factor to which causal efficiency was attributed is the actual cause in the case in question. Attention may have been drawn to the factor suspected to be causal by the uniformity of our observations that among the varying antecedents the one factor was always present. In that case the Method of Agreement would lead us to suspect the cause. If, then, we

are able to eliminate that factor while the other factors constituting the antecedent remain, we feel sure that none of those other factors is the cause; but only the one the absence of which is marked by the absence of the event. Of course the Method of Difference may often be used where observation without experiment discloses a fortunate situation in which the event at one time occurs with all antecedent factors present, and at another time does not, with all factors present save one. But as a rule it is only by experiment that we can secure a situation of this kind. Hence the Method of Difference is the great experimental method.

Mill states the formula for the Method of Difference in the following form, known as the Second Canon: "*If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former, the circumstance in which alone the two instances differ is the effect or the cause, or an indispensable part of the cause, of the phenomenon.*"

The following statement, though no shorter, may make the meaning clearer to any who find difficulty in understanding the one given above: *If upon comparing a situation in which the phenomenon under investigation occurs with one, otherwise the same, in which it does not occur, we find that the attendant circumstances differ solely in the presence of a given factor in the one case and its absence in the other, that factor may be regarded as causally connected with the phenomenon in question.*

The relation between the two methods is clearly stated in the following quotation: "*In the method of agreement*

a number of instances are taken agreeing only in the possession of two circumstances—the cause and the effect element common to them all. In this method (that of difference) *only two instances are taken, and they must be precisely alike, with the one exception—the presence of two circumstances in one, that is, the cause and the effect elements, and the absence of the same in the other. In the method of agreement we compare the various phenomena to note wherein they agree; in the method of difference, we compare the two phenomena to note wherein they differ."* [1]

108. EXAMPLE OF THE METHOD OF DIFFERENCE.—Suppose we enter a school-room and observe that the children are inattentive to a recitation that is in progress. We move around quietly toward the rear of the room and lower the windows slightly. The inattention vanishes. The circumstances have been changed in only one particular, viz., the absence of carbon-dioxide in harmful quantities. We naturally attribute the poor attention which we encountered on first entering to the only factor which was present when the phenomenon of inattention was present, and absent when it was absent.

Another example is furnished in the common experiment with the watch and the air-pump. Place a watch under the receiver of an air-pump and at a given distance one hears it ticking. Now let some one exhaust the air, and at the same distance the ticking is no longer audible, proving that the presence of air is the cause of the sound's reaching the ear. In this case we have the phenomenon to be explained occurring with a certain set of antecedent conditions. Remove one of those con-

[1] Hibben, *Logic, Deductive and Inductive*, p. 238.

ditions only, viz., the presence of air in the receiver, and the phenomenon does not occur. This experiment is analyzed in the following form:

	ANTECEDENT			CONSEQUENT
	OBSERVER	WATCH	AIR	
1st instance:	10 ft. off	ticking	present	sound heard
2d instance:	10 ft. off	ticking	absent	sound not heard

The following general formula stands for any experimental problem attacked by the method of difference:

ANTECEDENT	CONSEQUENT
A B C D	X Y
A B C	X

Of course there must be some consequence of the presence of any set of antecedent factors. Here X represents this indefinite consequence that lies outside the province of the given inquiry. X has with it Y when D is present with A B C; but when D is absent X occurs without Y. Hence D is proved to be the causal factor responsible for the occurrence of Y.

It should be noticed that this method depends for its application upon the investigator's ability to change only one factor at a time. Oftentimes this cannot be done, and even when done is usually accomplished by bringing about an artificial condition. Hence, as already noted, it is primarily an experimental rather than a purely observational method. It is used most effectually in physics and chemistry. In biological investigation it is not so safe because any variation of a single causal factor is liable to involve other obscure

changes that the investigator cannot easily foresee and calculate. It has been used, however, with very good results to determine causal connection in one branch of biological science—medicine. The effect of different kinds of remedies and modes of treatment has been studied by varying one causal element at a time and noticing the effect produced. In this way the effect of various low forms of life on the higher animals has been determined with remarkable accuracy.

109. JOINT METHOD OF AGREEMENT AND DIFFERENCE.—We have seen how the method of agreement is a plan for systematic observation and analysis of the factors antecedent to a phenomenon, and how it discloses its cause with a high degree of probability. We have seen also how the method of difference gives practical certainty as to the cause, but is mainly an experimental method, useful where the situation allows the experimenter to vary only one factor at a time, and consequently of limited applicability. We are next to consider a combination of the two methods hitherto noticed, which combines the applicability of the one with the certainty of the other. The Joint Method of Agreement and Difference permits us to have recourse to observation without experiment, if experiment is not available in a given situation. And at the same time it permits us to compare cases where the phenomenon is present with the antecedent factor under investigation with cases in which the phenomenon and antecedent are absent. It is especially useful as a method of record, where data are kept respecting observations that cover

long periods of time. Yet there is nothing to preclude its use as an experimental method also whenever the case favors experiment.

There are really three steps in the method—the first an application of the method of agreement, the second the same method, and the third the comparing of the two sets of results by the method of difference. First, then, the investigator collects a set of (positive) instances where there is a given effect and the factors making up the several combinations of antecedent circumstances all differ save in one factor (the usual method of agreement). Then he collects a set of (negative) instances where the effect under investigation is absent, and where the factors making up the antecedent circumstances are similar to the positive set, save that they agree only in the absence of the factor which was present throughout the first set of instances. Then the conclusions respecting the two sets of instances (the first agreeing only *in the presence of the effect under investigation and one antecedent factor*, and the second agreeing only *in the absence of the effect under investigation and the same antecedent factor before found uniformly present*) are compared by the method of difference. This makes us conclude that in a number of cases of the occurrence of the effect a certain factor was uniformly present, and in a number of cases of the non-occurrence of the same effect, the same factor was uniformly absent. Hence we hold that factor which is always present when the phenomenon occurs and always absent when the phenomenon does not occur to be the cause.

Mill states the canon (Third Canon) as follows: "*If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect or the cause, or an indispensable part of the cause, of the phenomenon.*"

This might be described as a process of comparison by the method of difference of sets of positive and negative instances, the first set agreeing solely in the presence, and the second set agreeing solely in the absence, of the phenomenon and antecedent circumstance under investigation.

110. Example of the Joint Method.—The following passage,[1] describing Darwin's experiments upon the cross-fertilization of flowers, furnishes an excellent example of this method:

> *Mr. Darwin, in his experiments upon cross and self fertilization in the vegetable kingdom, placed a net about one hundred flower heads, thus protecting them from the bees and from any chance of fertilization by means of the pollen conveyed to them by the bees. He at the same time placed one hundred other flower heads of the same variety of plant where they would be exposed to the bees, and, as he observed, were repeatedly visited by them.*
>
> *Here we have the two sets of instances, in one the flowers accessible to the bees (positive), and in the other, not accessible (negative).*
>
> *He obtained the following results. The protected flowers failed to yield a single seed. The others produced sixty-eight grains' weight of seed, which he estimated as numbering* 2720 *seeds.*

[1] Hibben, *Logic, Deductive and Inductive*, p. 253.

In reading the above description the student must remember that the order of narration in the first paragraph does not imply the logical order of regarding the experiment. The problem is to find the agency that produces the fertilization of the flowers of the species of plants investigated. Before experimenting Darwin was not fully satisfied whether the pollen of a given flower fertilized the ovules of the same flower, or whether an external agent transported the pollen of one flower to the stigma of another (cross-fertilization), though he suspected that the latter was the case. He proceeded to use the joint method by taking a hundred instances where the factor in question (access of bees to the flower heads) was in evidence, and observed that the effect (production of seed, which evidenced fertilization) followed. From this use of the method of agreement he concluded that when bees have access to flowers of the kind in question they are fertilized. Then he took a hundred instances where the factor in question (access of bees to the flower heads) was not in evidence, and observed that the effect (production of seed, evidencing fertilization) did not follow. By the method of agreement employed upon these (negative) instances he concluded that when bees (or other external agency) have not access to flowers of the kind in question, they are not fertilized. Finally he compared these conclusions by the method of difference, and reached the final conclusion that fertilization of the plants in question depends upon some external agency which produces cross-fertilization by transporting the pollen of one flower to the pistil of another.

We may further illustrate this method by another and more familiar example. Suppose a person is subject to indigestion following breakfast. He will naturally attribute the condition to some food he has eaten. He decides to observe the consequences of various combinations of food with a view to determine what one he must give up to avoid unpleasant consequences. The first breakfast observed he eats oatmeal, boiled eggs, rolls, and coffee. The second breakfast observed he eats grape-fruit, lamb chops, bread, and coffee. The third breakfast observed he eats buckwheat cakes, sausage, coffee, and maple syrup. The fourth breakfast observed he eats hot biscuit, honey, a wheat breakfast food, coffee, and oranges. In each case he suffers from indigestion. Examining his dietary he notices that coffee is the only invariable factor, whence he concludes by noting the agreement that it s the cause of the indiges ion. But we will imagine that he is addicted to the use of coffee and dislikes to give it up unless convinced beyond the shadow of a doubt that it is the cause of his trouble. To settle the question to his complete satisfaction he decides to observe the results of doing without his cup for a few mornings. If all the instances of abstaining from coffee agree in the absence of indigestion after breakfast, he can no longer doubt that coffee does not agree with him. For he has compared by the method of difference the results of using the coffee with the results of not using it, and finds that when he uses it he suffers the annoyance of indigestion, and that when he does not use it the unpleasant symptoms disappear.

The following formulas represent in an abstract manner any possible case of an investigation by this method:

EFFECT KNOWN; CAUSE TO BE DETERMINED

Positive instances

ANTECEDENTS	CONSEQUENT
A B C D	E^1
A F G H	E^2
A I J K	E^3

The method of agreement shows A to be the probable cause of E.

Negative instances

ANTECEDENTS	CONSEQUENT
—A X	—E L
—A Y	—E M
—A Z	—E N

The method of agreement used upon these negative instances shows that they are all alike in the absence of A and of E (indicated by placing the minus sign before each letter). It should be explained that the investigation would be limited to cases where we should think that the effect in question, represented in the formula by E, might occur owing to the nature of the antecedents. We let any such group of antecedent factors as might possibly come within the scope of our investigation be represented in the several instances by X, Y, and Z. Each group is characterized by the one uniform peculiarity of not having A as a factor. In the same way something must follow the occurrence of X, Y, and Z. We let this consequent which lies apart from the interests of our inquiry be represented in each instance re-

spectively by L, M, and N, each having the sole common characteristic of the absence of the effect we are looking for, E.

As the last step, our mind compares the results of the two agreements, one that when A is present E uniformly follows, and the other that when A is absent E uniformly fails to follow, and concludes finally that A causes E, as is shown by the table representing the last step.

Antecedent factor C present, consequent E invariably follows.

Antecedent factor C absent, consequent E never follows.

111. Method of Concomitant Variations.— Modern science has progressed mainly where it has succeeded in app ying mathematical computation to the facts under consideration. Wherever it is possible to number, measure, or otherwise reduce our facts to numerical comparison, we succeed in getting command of them in a way which makes for precision and exact knowledge. The method of Concomitant Variations is used to determine quantitative relations between the elements of the causal relation. It aims to show how much a given rate of variation in the cause will change the effect either directly or inversely. The term concomitant means accompanying. Hence the phrase refers to changes in the cause and effect accompanying each other in some definite ratio. Some form or other of this method has been particularly fruitful in the branch of biological science which considers the interdependence between changes in the nervous system and

changes in mental states, viz., physiological and experimental psychology. In general, the method is useful in any sort of investigation in which it is possible to record variations between the antecedent and consequent, and reduce such changes to at least a rough computation. The variation may be either direct or inverse. It is amply illustrated by the numerous laws of physics which embody the statement of either a direct or inverse ratio.

The canon (Fifth Canon) is formulated by Mill in the following language: "*Whatever phenomenon varies in any manner, whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.*"

The following statement formulates the same thought more briefly: *Whenever two factors vary together uniformly we have in this fact a proof of causal connection.*

112. Examples.—Concomitant variation shows friction to be the cause of heat when one observes that the temperature of two sticks rubbed together rises in proportion to the energy put into the rubbing, or that a piece of iron grows warm as more blows rain down upon it. That cold causes contraction is shown by the fact that as the temperature decreases the length of the rails decreases (as shown by the gap at the junction point). Another illustration is Boyle's Law for the volume of gases under pressure, which states that the volume of a gas varies inversely as the pressure to which it is subjected.

The following formula is a general expression for a problem in concomitant variations:

ANTECEDENT	CONSEQUENT
1 C	1 E
2 C	2 E
3 C	3 E

The coefficient numerals, 1, 2, etc., indicate the quantitative value of each element in the causal couple.

113. METHOD OF RESIDUES.—The term residue as here used means remainder. The principle underlying the method is this: where several factors contribute to the effect in question we can find out how much a factor that is not directly determinable contributes by subtracting the known contributions of the several other factors.

The first use to which this method is put by scientists is to determine what proportion of any effect under investigation—such effect being known as the product of combined factors—is due to a hitherto undetermined factor, when the amounts due to the several other factors have been previously determined.

The second use of the method is when a minor residual effect is noticed in the course of some main investigation. Oftentimes such a "left-over" factor in the effect is ignored for a long time. Finally it gets the attention of some scientist as something not hitherto explained but requiring investigation. The main subject of inquiry may be temporarily abandoned and an exhaustive research begun to determine the cause of the unexplained part of the effect (called a "residual phenomenon"). In this way some of the most important dis-

coveries in the history of science have been made. For oftentimes these inconspicuous residual phenomena prove vastly significant and lead to new principles of nature. The term residual phenomena was first used by Herschel the astronomer.

Mill states the Fourth Canon as follows: "*Subduct* (take away) *from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.*"

114. Examples.—First use: Suppose that a farmer sells a load of hay and wishes to know what weight he is to charge the purchaser. He puts the wagon loaded with hay upon the platform and finds the gross weight of the hay together with the wagon. Then after he has unloaded the hay he returns to the scales and weighs the wagon, and subtracts its weight from the gross weight of hay and wagon in order to find the net weight of the hay. In this case the cause of the first weight is the wagon and the hay together. Then the farmer easily ascertains what part of this effect is attributable to the wagon, and after subtracting this ascertained factor he knows that the remaining effect is due to the hay alone.

Second use: A classical illustration of research into the significance of residual phenomena is furnished by the discovery of the planet Neptune by the astronomers Adams and Le Verrier, each working independently of the other. Certain unaccountable irregularities in the movement of the planet Uranus had already been observed. The influences of none of the known stars could account for them. Uranus behaved as though

some body beyond its orbit were disturbing its movements. Adams and Le Verrier calculated by mathematics where such a body as would account for the unexplained effect (residual phenomenon) should be found. Directing their telescopes to that point in the heavens at an opportune time, they discerned the planet Neptune, which accounted fully for the aberrations of Uranus.

The following paragraph gives a clear presentation of the importance of residual phenomena:

> *When, in an experiment, all known causes being allowed for, there remain unexplained effects (excessively slight it may be), these must be carefully investigated. . . . It is here, perhaps, that in the present state of science we may most reasonably look for extensions of our knowledge; at all events, we are warranted by the recent history of natural philosophy in so doing. Thus, to take only a very few instances, and to say nothing of the discovery of electricity and magnetism by the ancients, the peculiar smell observed in a room in which an electrical machine is kept in action was long ago observed, but called the "smell of electricity," and thus left unexplained. The sagacity of Schönbein led to the discovery that this is due to the formation of ozone, a most extraordinary body, of enormous chemical energies, whose nature is still uncertain, though the attention of chemists has for years been directed to it.*[1]

The method of residues is, like that of concomitant variations, a method of determining quantitative relations. The problem that it attacks is that of discovering how much of the effect is due to one factor among a group of antecedents that have causal value.

[1] Thompson and Tait, *Elements of Natural Philosophy*, Vol. I, pp. 113 *f*.

Both methods are in reality special aspects of the great experimental method, that of difference. For in concomitant variations we note the difference between any effect produced by a given amount of the cause and a similar effect produced by a greater or less amount of the cause. Likewise in finding residues we determine the difference between the effect due to that part of the cause whose value as a contributing factor has previously been determined and the total effect as it presents itself with the undetermined factor operative.

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Welton, *Manual of Logic,* Vol. II, Bk. V, Ch. V.
Mill, *System of Logic,* Bk. III, Chs. VIII–X.
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Aikins, *The Principles of Logic,* Chs. XXVI–XXIX.
Venn, *Empirical Logic,* Ch. XVII.

REVIEW QUESTIONS

1. *What do you understand by the antecedents of a phenomenon? by the consequents?*
2. *Where is the cause of a phenomenon to be looked for? the effect?*
3. *What is each cause in relation to its antecedents? each effect in relation to its consequents?*
4. *What are causal couples?*
5. *What does scientific explanation aim to do?*
6. *What do Mill's methods really do?*
7. *Why are they part of induction?*
8. *How many irreducible ways are there, according to Mill, of analyzing causal couples so as to find causes and effects?*
9. *For what is the method of agreement used?*
10. *Why does the method often fail?*
11. *Why are laws discovered by scientists called empirical?*
12. *Give Mill's formulation of the First Canon.*

13. *Givc Jevons's and Creighton's formulation of thc First Canon.*

14. *Is there any differenee, so far as the method is eoneerned, between discovering a eause and an effeet?*

15. *State and illustrate thc three conditions under whieh the method of agreement is untrustworthy.*

16. *Why is the method of agreement mainly used in observation?*

17. *Give the abstraet formula for the method of agreement used to analyze the eausal factor out of the antecedents; for the effect.*

18. *Why is the method of difference especially adapted to experiment?*

19. *Give the two formulations of the Seeond Canon.*

20. *What is the relation between the method of agreement and that of difference?*

21. *Givc the abstraet formula for the method of difference used for the eause; for the effeet.*

22. *What are the limits of the applieability of the method of differenee?*

23. *In what scienees has it provcd most useful?*

24. *What advantages has the joint method over either of the others so far studied?*

25. *What kind of observations can be best trcated by this method?*

26. *Name the three steps in the method.*

27. *State the Third Canon.*

28. *Give the abstraet formula.*

29. *In what way has modern scicnce progressed most?*

30. *What is the meaning of "coneomitant"?*

31. *What does the method of coneomitant variations aim to show?*

32. *What sciences have employcd the method with good results?*

33. *What two types of variation are there?*

34. *State the Fifth Canon. Givc the briefer statement.*

35. *Give the formula for direet and inverse variation.*

36. *What principle underlies the method of residues?*

37. *What are the applieations of the method?*

38. *State the Fourth Canon.*

39. *What do you understand by residual phenomena?*

40. *Show why the method of residues is also quantitative.*

41. *What is the relation of the method of eoneomitant variations to the method of difference? Of the method of residues to the method of difference?*

EXERCISES ON CHAPTER XVII

1. Imagine some situation or investigation where the method of agreement should be used to prove the cause. Analyze and tabulate the factors. Then symbolize the inquiry by letters.

2. Follow the above direction in an imaginary investigation to determine the effect.

3. Construct an imaginary investigation where the method of difference should be used to determine the cause. Analyze the factors (as on p. 241) and construct a proper table. Then symbolize the inquiry by an abstract formula of letters.

4. Construct an imaginary instance where the joint method should be used to prove the cause. Then give an abstract formula that is adapted to describe it.

5. Cite some law of variation from some science you have studied (Physics furnishes excellent examples), and be prepared to show how it is illustrative of the method of concomitant variations.

6. Construct an imaginary instance where the method of residues may be used to determine what contribution to the effect a joint causal factor makes.

7. Analyze the following examples, name the method which they illustrate, and write formulas adapted to them:

> (*a*) *A core of soft iron wound with copper wire is suspended over a pile of iron filings. One end of the wire is connected with a pole of a galvanic battery. Upon completing the circuit by connecting the other end with the other pole the filings leap to the adjacent end of the iron core and cling there.*
>
> (*b*) *From* 1845 *to* 1848 *Ireland suffered from a terrible famine, which reached its climax in the latter year. During this period there was a very noticeable increase in agrarian crime, which was three times as common in* 1848 *as in* 1845. *Thereafter it diminishes with the return of better crops until it was only fifty per cent. greater in* 1851 *than in* 1845. (*Adapted from Hyslop.*)
>
> (*c*) *Sachs found that the leaves of plants containing chlorophyl and absorbing carbon-dioxide formed starch when exposed to the light and ceased forming it when re-*

moved from the light. He concluded that starch results from the decomposition of carbon-dioxide in chlorophyl under the influence of light. (*Adapted from Creighton.*)

(*d*) *During the months of October and November,* 1907, *there was great stringency in the American money market, leading to a general retrenchment in business. It was noticed that the outgoing steamers that left the port of New York in late November and early December had unusually large bookings for steerage passage to Europe.*

(*e*) *Tyndall opened twenty-seven sterilized flasks in pure Alpine air and found no signs of putrefaction. He later opened twenty-three in a hay-loft and only two showed no signs of putrefaction after three days.* (*Adapted from Creighton.*)

(*f*) *During a recent trial intended to place responsibility for a railroad wreck, the following facts were submitted in evidence: At the time of the wreck a train of* (*say*) *seven cars was drawn by two electrical engines. The controller was set at "series-parallel"* (*implying about three-quarters full speed*). *The train left the track in rounding a curve. Later a test was made over the same track by State officials, with a duplicate of the wrecked train, with the controller at series-parallel, and the train did not jump the track. Testimony was introduced proving that the outside rail was double-spiked at the curve during the time intervening between the accident and the official tour of inspection.*

(*g*) *Goldscheider showed that perception of movement is mainly due to pressure-sensations from the inner surface of the joints. He had his arm held so that the surfaces of the joints were pressed more together, and found that he could distinguish a smaller movement.* (*Adapted from Creighton.*)

(*h*) *The deeper a person descends into a mine the higher rises the column of mercury in the thermometer. The higher a person ascends a mountain the lower falls the column of mercury in the baromcter.*

(*i*) *"The bull-frog just from the mud, or from some place of concealment in deep water, is so dark-colored that he is nearly black. Experiments prove that light has much to do with these changes of color; at least, that these changes in color take place with changes in light, when temperature*

and moisture conditions remain the same." (Dickerson, *The Frog Book*, p. 230.)

(*j*) (Description of the exposure of a spiritualistic séance.) "*The two females were then seated upon two chairs placed near together, their heels resting on cushions, their lower limbs extended, with the toes elevated and the feet separated from each other. The object in this experiment was to secure a position in which the ligaments of the knee-joints should be made tense and no opportunity offered to make pressure with the foot. We were pretty well satisfied that the displacement of the bones requisite for the sounds could not be effected unless a fulcrum were obtained by resting one foot upon the other or on some resisting body. The company, seated in a semicircle, quietly waited for the 'manifestations' for more than half an hour, but the 'spirits,' generally so noisy, were now dumb. . . . On resuming the usual position on the sofa, the feet resting on the floor, knockings very soon began to be heard. . . . The conclusion seemed clear that the Rochester knockings emanate from the knee-joint.*" (Quoted by Podmore, *Modern Spiritualism*, Vol. I, p. 184.)

(*k*) "*In* 1893 *Lord Rayleigh undertook to determine the density of nitrogen with all the accuracy of present-day science. To his astonishment, he discovered that nitrogen from the air and nitrogen from chemical compounds did not weigh the same. The difference was small but exasperatingly constant. Out of this curious anomaly arose the discovery of a new and hitherto unsuspected element of the air which had been weighed as nitrogen and considered as nitrogen by all preceding chemists. This new element was named argon and it constituted nearly one per cent. of the air we breathe. Subsequently this 'argon' was discovered to be itself impure, and from it were isolated four other elements: helium, neon, krypton, and xenon.*" (Duncan, *New Knowledge*, p. 36.)

CHAPTER XVIII.—THE FALLACIES OF INDUCTION

115. Character of the Fallacies of Induction—We are liable to certain habitual forms of error in pursuing inductive investigations. These forms of error are known as the inductive fallacies. Some of the common errors already met with in deduction will meet us again here. Others are peculiar to induction. They are all psychological in their nature. They are ways in which the human mind is likely to err when it is especially striving to get at the truth. They are incident to the several kinds of functioning of the mind. Some occur in perception, some in the use of the imagination; others in forming judgments of experience, and still others in conception. Accepting the usual classification of the mind's functioning, we may readily distinguish the following great categories of inductive fallacies.

I. Errors of perception.
II. Errors of memory.
III. Errors of imagination.
IV. Errors of apperception.
V. Errors of conception.
VI. Errors of judgment.

116. Errors of Perception.—We are here considering perception employed in the service of science. Perception involves the use of the various senses, especially

those of sight and hearing, and the interpretation of the sense-facts introduced by the senses into the forms of the various objects that make up the material universe. We shall assume that the sense-organs are capable of doing their work of furnishing the sense-data. Then errors of perception will be limited to the use we choose to make of the senses, and particularly of sight, which is most prominent in observation.

The first error of perception arises from failure to take in the entire field of observation. In this way some fact having a vital bearing upon the investigator's problem may be overlooked. It not infrequently happens that some obscure part of the object is ignored, when this part, if noticed, would have caused an entirely different interpretation of the phenomena. Even skilled scientists are sometimes guilty of this error. The only way to guard against it is to cultivate the virtue of patience. It is usually the overhaste of the observer that is responsible for carelessness in this regard. A scientist, eager in the search for facts confirming a theory, is prone to select favorable aspects from the field of observation. If they are found, he hastens on without careful observation of other parts of the field. Among such neglected parts may lie facts that would upset his theory. But the most careful scientists will look over the entire field without slighting any part, making truth, and not the establishment of a pet theory, the goal of their endeavor.

The second error of perception to be noticed is the tendency of the investigator to thrust his own mental states into the objective field. In general one sees what he wants or expects to see. What the psychologists call

expectant attention accounts for many of the strange experiences to which people of undeniable veracity bear witness. Let the mind be charged with an interesting and absorbing idea, and the tendency is to find in our sensations an object corresponding. The fear-haunted lad who has to pass a graveyard late at night is already far on his way to seeing ghosts before he reaches the dreaded spot. This tendency often amounts to an actual sensing of the thing corresponding to the idea. In some cases of mental disease the subjective states take the form of hallucinations that have for their unfortunate victim a veritable sensuous reality. Even under normal conditions we tend to perceive in accordance with the dominant group of ideas lying within the field of consciousness. Now the scientist always brings an organized group of dominant ideas to his problem. Added to this are numerous habits of thought that influence his perceiving. These ideas and habits are essential to all fruitful use of the senses, and yet they may be also a pregnant source of danger. They are necessary because they give his work initiative, energy, and direction. They are dangerous because unless he be constantly on his guard they will bias his observations and vitiate the scientific accuracy of his results. If he comes to his field of observation surcharged with such a group of ideas, he may project them as sense-data into the objective field and take for objective reality what is merely a subjective prepossession. The only way to prevent this kind of error is to test by repeated examinations of the field under investigation whether the supposed facts are really there. The scientist who puts

truth above his ambition to win a name as a discoverer will be likely to avoid this pitfall. It is possible to form a habit of suspicion toward preconceived ideas and theories. This kind of habit will be invaluable in preventing the intrusion of the subjective mental states into the objective field.

117. Errors of Memory are among the most potent sources of erroneous induction. They are of every-day occurrence, and are thus very insidious. Defective retentiveness is difficult to correct because it is largely dependent upon the physical structure of the nervous system, and over this it is impossible to exercise effective control. Yet much of so-called defective memory is due to careless habits of attention while an investigation is in progress, and such habits may be counteracted by better ones. The investigator should approach his problem under the best physical and mental conditions, and good sensing is likely to result. If exact and vivid sense-percepts are secured, definite and lasting memory-images are the natural consequence. Power to recall our memory-images just when they are wanted is largely dependent upon the way our ideas are organized through association. If, when a percept is received, its relations to other experiences along the same line are definitely determined, an organized group of images comes to be a permanent possession of the mind. If a new experience calls up one of the group, an escort of related images comes to the foreground. The organization should be secured rather by association by similarity (and difference) than by contiguity. For this affords a more rational basis of organization and naturally en-

hances the chance of useful recall. But even when all is said and done the memory is a treacherous servant. Where such important issues are at stake as in science, it is usually best to make a written record of the facts that are to be the basis of the induction. Since words are prolific sources of error, it behooves the recorder to make his record brief and technical, thus economizing time and minimizing the chances of ambiguity.

118. Errors of Imagination are closely akin to those of memory. Memory-images furnish the material out of which the imagination builds its products. The great danger is in unrestrained fancy. If properly used, imagination is of the greatest service. It has already been shown that the imagination must conceive the plan for any line of investigation. It furnishes the hypotheses that are essential to direct scientific research. But it is necessary that the imagination should keep close to facts. Uncontrolled fancy is liable to vague guesses rather than accurate hypotheses constructed upon a foundation of past experience. While the poet and the romancer may indulge their fancy unrestrained, the scientific worker must harness his imagination to fact. He must continually test his hypotheses by referring them to real events and circumstances. If they do not measure up to the requirements of fact they must be summarily dismissed. The scientist must also draw the line between facts of observation and the ideas framed by the imagination from those facts. Minds otherwise brilliant often fall into sorry straits by confusing theory with reality. So far as concerns accurate scientific results nothing could be more fatal. The

most successful scientists pursue their researches under the guidance of theory, but never do they allow even the most alluring theories to stand in opposition to the data given by observation. The imagination is most serviceable when it weaves the facts of observation into an organized conception that has internal consistency. In such a construction of the imagination the facts observed furnish the framework, and the mind through its imaging power fills in the gaps where facts are wanting. But this contribution of the imagination must be in harmony with the facts. The worst error to which imagination is liable comes from keeping aloof from the teachings of observation. Some people's minds are so constituted that they always run on away ahead of the facts or else twist the facts to suit their idle fancy. The best caution that we can give is to urge the student of logic to be the master, not the slave, of his imagination. Press it into service wherever needed, but see that it introduces no inconsistencies and sticks close to actual experience.

119. Errors of Apperception.—These errors arise from a tendency to be too much biassed by our past experience and habits of thought. Were it possible for a human being to do it, there can be no doubt that he would be fairest to a new fact of experience by dismissing from his mind all prejudices arising from previous experiences of a similar nature. But this is not possible to minds constituted as ours are, for the new experience would have no meaning for us if utterly unrelated to our previous life. All real human knowledge depends upon interpreting the new in the light of the old. This being

the case, we cannot *know* facts in isolation from others. Hence apperception, by which is meant nothing more than interpreting the novel elements of our experience in terms of the old, habitual, and familiar, is of the greatest importance in learning and, indeed, in all reasoning processes. But apperception, like the other useful functions of the mind just considered, may lead us astray. Our business just here is to find wherein the danger of apperception lies in order that we may secure its benefits and avoid its pitfalls. When the investigator is examining a field to find just what is presented to his senses and no more, he would be wrong to let his thoughts and ideas about the subject-matter in question creep in so as to be mistaken for and reported as an actual part of what he discovers. It is in the subtle tendency of these previous thoughts and ideas to project themselves upon the objective field and be mistaken for realities that the trouble lies. Perhaps it is never wholly possible to avoid this error. For it is a universal trend of the human mind to put its own interests into the things about it. But we can at least be on our guard against the tendency, and apply tests to determine whether our observations are true to the object as it is or are tinged by our own mental prepossessions.

120. Errors of Conception.—These mistakes arise in forming our concepts or general notions about things and experiences. The mind leaps forward in its overhaste to generalize. Oftentimes the experiences are too narrow to be a safe basis for a general idea. Persons whose destiny is to lead narrow lives are very prone to interpret the whole world by the petty circumstances of

their humdrum existence. And even those who have had better advantages and have seen more of the world find it hard to take wider views than their personal experience suggests. Again, some people are temperamentally so constituted that they cannot appreciate the point of view of others. Hence arise bigotry in religion and intolerance of antagonistic opinions. These persons of narrow experiences or narrow tendencies are like prisoners in a dungeon from which only one small window opens upon the outer world. In time they come to believe that what they see is all there is to see. Now all such persons are falling into the most common of all inductive fallacies, hasty generalization. From a narrow field of experience their minds sweep on to the most universal conclusions. They are shut into their own little mountain valley by insurmountable peaks, and they believe that all the great outlying world consists of mountain valleys just like their own. Hasty generalization is what we express by the colloquial phrase "jumping at conclusions." The fallacy is so common that no matter how we are on our guard we cannot altogether escape it. We *will* jump at conclusions in spite of ourselves. But we may get into the saving habit of weighing evidence carefully before reaching a conclusion, in order to see whether there is sufficient ground for the generalization we are making.

Another form of this error which is closely allied to that just noticed is the failure to note the boundary-line between classes, and the tendency arising out of this confusion between species, to apply a generalization derived from a study of one class to a related class to

which it does not apply. This is really a consequence of careless classification, and the best way to avoid it is to take time to classify the facts with the utmost care.

121. Errors of Judgment.—These fallacies arise in interpreting the facts observed. They may be distinguished into the following varieties:

1. Taking associations between experiences to be causal connections when they are merely coincidences.

2. Emotional or prejudicial preference, by which is meant a stubborn persistence in finding a certain unreal meaning in our facts because it suits us better to look at them from this angle.

3. Misjudgment due to common human frailty and the unavoidable limitations of our knowledge.

The mind associates events either because they have happened together in place or time (association by contiguity) or because some real ground of relation such as likeness or causal connection has been detected. We frequently take the former association for the latter, as when the farmer declares that a phase of the moon is the cause of rain, on the flimsy ground that the two facts chanced to occur at the same time.

Our feelings influence our interpretation of facts, making us judge as we like to rather than with strict fidelity to the facts in question. Should the facts dispute our firmly rooted prejudices, we ignore the facts. Science makes little headway under these circumstances. Bacon, the English philosopher, tells us:

The human understanding resembles not a dry light, but admits a tincture of the will and passions which generate their own systems accordingly; for man always believes more readily

that which he prefers; his feelings imbue and corrupt his understanding in innumerable and sometimes imperceptible ways.[1]

The mere fact that we are human makes us heirs to certain kinds of fallacies that can never be totally avoided but against which we may, nevertheless, be on our guard. Bacon has mentioned these under the figurative name of "idols"—"Idols of the Tribe," "Idols of the Den," "Idols of the Market," and "Idols of the Theatre." The first refer to the influence upon our judgment of the necessity of measuring everything from the human point of view, as when we look upon all lower animal life from the stand-point of its usefulness to ourselves. The second are the peculiar warpings of judgment arising from the special experience of each person, as instanced in the minister's regarding the world of his fellow human beings as a problem of religious salvation, or the teacher's looking upon children as a problem in instruction. The cut of the cloth is upon each one of us, and our trade in life distorts and colors all our social relationships. The third type are those errors that arise from the use of such treacherous symbols as words as the vehicles of thought. We have already discussed them under ambiguity of terms. Last are the biasses due to some form of authoritative tradition, either religious, philosophical, or matter of unverified common belief.

No pretence is made that the above list of inductive fallacies is complete. Errors incident to induction are very subtle and resent accurate classification. But we have before us all of the most important general varie-

[1] *Novum Organum,* Bk. I, Aphor. XLIX.

ties. The fulness of the list is sufficient warning that the person who is ambitious for accurate scientific knowledge must be ever alert in guarding against their intrusion into his problem. Admitting that many of them cannot be wholly avoided, still much may be done by watchfulness to develop unprejudiced habits of mind. Such habits once formed, the most serious source of error is removed, and the minor ones will take care of themselves.

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REVIEW QUESTIONS

1. *What is meant by a fallacy?*
2. *Name the main kinds of inductive fallacies.*
3. *Name the more important senses employed in perception.*
4. *What is the first error of perception and what its consequence?*
5. *To what is the second error of perception due?*
6. *What is the best way to prevent this error?*
7. *In how far may errors of memory be corrected?*
8. *How may they best be corrected?*
9. *To what are errors of imagination closely related?*
10. *Wherein lies the chief danger in the use of imagination in scientific investigation?*
11. *What is the best way to correct the vagaries of unrestrained fancy?*
12. *In what do errors of apperception have their source?*
13. *How may the scientist avoid the errors of apperception?*

14. *What is the first kind of error of conception?*
15. *How may it be avoided?*
16. *What is the second kind of error of conception and how avoid it?*
17. *Name the three varieties of errors of judgment.*
18. *What are the "Idols of the Tribe"?*
19. *What are the "Idols of the Den"?*
20. *What are the "Idols of the Market"?*
21. *What are the "Idols of the Theatre"?*

EXERCISE ON CHAPTER XVIII

Try to imagine or cite an example of each kind of inductive fallacy mentioned above.

PART VI.—LOGIC AND EDUCATION

CHAPTER XIX.—LOGIC AND THE ORGANIZATION OF KNOWLEDGE

This chapter will attempt to cast a backward glance upon the subjects treated in the foregoing pages. It will consider the logical processes in their organic unity, and thereby present the science of logic as a finished whole of interdependent parts.

122. How Logic May be Misconceived.—The student who puts to himself the question, What is the purpose of logic? may mean either of two things. If he merely means to repeat the silly query with which the indolent utilitarian greets most subjects that really discipline by calling forth genuine effort, his question is hardly deserving of a serious answer. But if logic seems to him hazy and unreal, and if he seeks genuine enlightenment as to why logic should seem to the beginner somewhat artificial and super-analytic, and yet lay claim to a place in his training for the critical mental adjustments of life, a serious consideration is due him. The best answer is to show him that the apparent artificiality of logic is only a seeming—the unavoidable consequence of presenting an articulate whole of thought in an analytical and piecemeal fashion.

Logic has to be studied step by step. The complex

function of reasoning is analyzed into its aspects by logical partition, and these several aspects are presented chapter by chapter. This may easily result in a type of fallacy very similar to hypostasis of abstractions (*cf.* p. 18). It might be given the name *hypostasis of analytic exposition.* The student of anatomy might just as easily succumb to this error were the tendency not checked by reference to the material facts presented by his own body. For just as a logic text has a chapter on terms, deduction, induction, etc., a text of anatomy has a section describing the circulatory system, the digestive organs, etc. The mistake that the anatomist escapes the student of logic is an easy victim to, because no material facts correct him.

123. Reasoning an Organic Process.—The most serious side of this error comes from the view of reasoning to which it lends color. The mind looks like a machine made of an aggregate of parts that work in series, not like the organism that it really is. To correct this misapprehension one needs only to study reasoning as an organizing activity of the mind. This will disclose the organic nature of rational thought and at the same time show the organic character of its product. It will convince us that any thought-activity involves in more or less completeness all the mind's powers of functioning, just as the normal action of the heart in one part of the body requires the healthy working of the nervous system in another, and vice versa. Thinking stands over all the subordinate activities of life much as a general does to his army. It is his duty to control the destinies of the forces under him, to foresee needs and

ends of adjustment, to survey the whole field of contemplated campaign, and to bring to bear upon such situations as may arise the widest knowledge both of the present factors and of past experience. Just as the general organizes his line of battle in view of circumstances that the plain soldier could not understand, so rational thought secures adjustment to intricate situations that completely baffle the lower and more mechanical activities of the nervous system. Interpreted by this analogy, thought is much more truly presented than when conceived as mechanical. Hence we may define reason as *the biological function which secures organization of experience so as to make it profitable in further adjustment.* And if it seems needful to distinguish logical thinking as a special type of the genus above defined, it may be done by adding the difference that such thinking is *fully conscious of the difficulty both of the organization of experience and its application to the problem of further adjustment, and employs special method to minimize the chances of inadequacy in both.* Hence logic is the *science of the methodology of organizing and applying experience.*

124. THE JUDGMENT AS AN INSTRUMENT OF ORGANIZATION.—Some aspects of judging were briefly discussed in a previous chapter.[1] We now need to review the subject in relation to the kind of truth which the several types of judgment assert. Briefly, we may deal with the matter as though all attitudes of thought could be adequately expressed in some one of the four types of proposition, A, E, I, O.

[1] See p. 65 *ff.*

I and O may be interpreted as forms of judgment subordinate to A and E rather than radically different from them. A little analysis will make this evident. I and O may have either of the following meanings: first, *some at least;* second, *some only.* In the former sense they indicate the reserve that the careful speaker uses when his experience is known to be incomplete, but up to date has shown no exceptions. They then correct one's proneness toward hasty generalization. In the latter sense they reveal the inadequacy of classification to meet the needs of the shifting phases of experience. Classes arise from the focus of attention upon relatively permanent and profound attributes. But things reveal themselves in constantly changing manifestations. Furthermore, the individuals of any class, though bound together by the generic attributes, are also differentiated into minor groups by the specific and individuating qualities. For these temporary aspects and minor groupings of things language does not always furnish class names. Hence we have to resort to the simple expedient of the particular judgment. The arrangement of experiences in permanent classes, though vital to the maintenance of a common fund of knowledge, is oblivious to the pressure of special interest in temporary phases and partial aspects of these permanent classes. The permanent class remains intact under the guardianship of its class name, but the individuals composing it are sorted out in all conceivable ways as convenience may dictate. Thus the second use of *some* in the I and O propositions responds to a pressing need of thought.

But in reality both these uses of the particular propositions point to judgments that are merely subordinate to the universals. *Some at least* will either occasion a new sub-class or later events will prove the judgment to be universally valid, the outcome being contingent upon whether or not fuller investigation of the field covered by the judgment reveals exceptions. *Some only* is a way of subdividing a class into smaller groups that are to all intents and purposes new sub-classes which may be predicated about universally.

Such makeshift classes may readily be arranged so as to subscribe to the formulas of universal judgment. So it is that after judgments about the class dogs have exhausted the generic attributes domesticable, barking, faithful, tail-wagging, etc., the analyzing activity of the mind focusses attention upon characteristics that are partial and accidental to the class as such. Color, size, shape of muzzle may all vary greatly throughout the class. This demands a redistribution of the individuals into smaller groups that allow these special attributes to be predicated.

Judgment therefore asserts that *some dogs are black* and *some dogs are not black.* But it puts the truth in an equally clear light to say, *All these dogs are black; All those dogs are not black.*

125. Function of the Universal Judgments.—Having shown how the particular judgments are special instances of the universal judgments, it remains to discuss the function of these latter. The A judgment is the mode of thinking a species under its genus. It represents mental synthesis. In affirming that men have religious instincts, I find my attention focussing upon the results of previous classification, the concept *man* used distributively, then shifting to the attribute *possessing religious instincts.* This attribute constitutes a phase of experience that may possibly be found elsewhere than in man. It absorbs attention as a special object of thought. It is thereby treated as a class-designation that embraces certain orders of experience. Man belongs under such a class as a species of manifestations of religious instinct. This is more clearly shown by an example in which we take classes that have become conventional and stereotyped bounds of experience. *Dogs are animals* is a judgment that relegates a definite class of experiences to another class of experiences having wider scope. It refers a species to its genus.

The E judgment, on the other hand, distinguishes between classes. And if its use is other than purely formal, if it is pregnant with meaning, it distinguishes between the species of a genus. For instance, the judgment, *spiders are not insects,* involves a connection in thought between spiders and insects to which we are led by their many similarities of appearance, and one

that is justified by the relation of both classes, conceived as species, to the same genus, *arthropoda*. Again, when we assert the judgment, *the Bretons are not of French blood,* we connect the Bretons with the French through the fact that they are under the same government. This is the genus which might readily enough suggest their racial identity. But our more accurate thinking reveals a difference that relegates the Bretons to one species of the genus, *those-acknowledging-the-sovereignty-of-France,* those of French blood to another.

Thus it appears that the universal affirmative judgment (A) refers a species to its genus, and is therefore synthetic and classificatory; and that the universal negative judgment (E) distinguishes between the species of a genus, and is hence analytic and dividing. The subordinate forms I and O represent thinking on the way toward A and E, or else they indicate temporary classes and are essentially A and E in a new guise.

126. THE MOODS AND FIGURES AS INSTRUMENTS OF THESE JUDGMENTS.—We shall now consider the conventional standards of deductive reasoning as instruments for developing these types of judgment. The only possibilities of combining judgments so that a new judgment emerges from their relationship (the only valid premises) are as follows: first, A and A; second, A and E; third, A and I; fourth, A and O; fifth, E and I (the order of the judgments is not here regarded). An inspection of these results shows that an A judgment, which functions to bring a species under its genus, is combinable with all other modes of judgment; an I, being an A partially completed, is combinable with two other

modes (A and E); an E, which differentiates species under a genus, is combinable with two other modes (A and I); while O, which combines both negation and particularity, is combinable with only one other mode (A). These results are just what we should expect from the fact that the A judgment synthesizes the various smaller aspects of experience into an ever-enlarging system, and thereby induces more inclusive unities upon the items that constitute our experience; that the I judgment performs the function of the A in less measure, and intimates the chance of new experiences offering exceptions or else suggests the inadequacy of a given classification to serve when a new aspect of the facts claims attention; that the E judgment analyzes anew and differentiates established classes into their species, and hence functions as a destroyer of systems; and that the O judgment has the double weakness of drawing attention to a partial aspect of experiences and also that of differentiating classes (possesses characteristics found in both I and E).

Now the ultimate aim of all uniting and systematization of experiences is a better understanding of the real nature of experience in the aggregate. And hence synthesis of the manifold items into an adequate system is the actual aim of knowledge. But it often happens that inadequate relations are suggested in the process of organization. Hence the need of analysis and of the negative judgment. But such a breaking down of classes as it effects is rational only on the ground of a further and more effective synthesis. And it seems evident that affirmative judgment of universal character is the ultimate aim of all thinking and reasoning. We

voice our sentiment in its favor in the claim that all satisfactory definition should be affirmative rather than negative. For defining relegates the things denoted by the defined term to their system.

Judgments attained through reasoning are the products of mediation. By this it is meant that they are reached only after the relation of one concept in the judgment to another concept is established. Expressing the same truth in terms of the organization of experience, it may be said that a phase of experience selected by attention for special study is frequently referred to its system not directly, but by establishing its relation to some other phase of experience whose relation to it is more directly apparent. Hence we may regard the syllogism as an *instrument for organizing experience through the accurate establishment of relationships determined by mediation.* This is evidently true of the affirmative syllogism, and it becomes true also of the negative syllogism if we regard it as temporarily disorganizing knowledge in order to pave the way for adequate organization ultimately.

Mediate thinking may occur in many ways. But the relation of the medium through which the reasoning takes place to the terms whose junction is sought in the resulting judgment admits of four variations, giving rise to the figures of the syllogism. An examination of the judgments as arranged in the figures proves interesting. Remembering that the conclusion either refers a species to its genus (A or I) or distinguishes between species of an implied genus, we note the following peculiarities of the reasoning in each of the figures.

First figure: the middle term is subsumed under (or differentiated from) the genus represented by the major term and includes (at least in part) the species represented by the minor term.

Second figure: the middle term includes (or excludes) the species represented by the major term and excludes (or includes) that represented by the minor term.

Third figure: the middle term is subsumed under (or differentiated from) the genus represented by the major term and is also subsumed under (at least in part) that represented by the minor term.

Fourth figure: the middle term includes (or excludes) the species represented by the major term and is subsumed under (or differentiated from) the genus represented by the minor term.

To sum this up: the middle term in the first figure may be a species in relation to the major term and is a genus in relation to the minor term; in the second figure it may be a genus to either term; in the third figure it may be a species under either term; and in the fourth figure it may be a genus to the major term and a species under the minor term.

These relations may be clearly apprehended by consulting the following diagrams. The devices used will be understood from the subjoined explanation.

M = major term; *m* = middle term; M = minor term (note difference in the size of M and M). The arrowhead indicates that the term following includes (represents a genus including) that preceding. The dash without the head indicates that the terms between which it is placed are not related as genus and species. Another way of showing the relations graphically is added for completeness. Here the symbols of the terms are placed so as to lie in a vertical line if they are related as genus and species, and

to lie out of the vertical line if they have not that relation. The caret ($\wedge$) shows that a term over its point is a genus of that under its opening. % means unrelated as genus and species. The moods are arranged adjoining the diagrams for the figures. It is to be understood that no distinction is made in any case between particular and universal judgments.

127. Functions of the Figures.—

FIRST FIGURE

There are two cases, one in which the middle term is a species of the major term, the other in which it is not; in both cases the minor term (wholly or in part) is a species of the middle term.

$$
\begin{array}{lcc}
m \longrightarrow \mathrm{M} & \mathrm{M} & \mathrm{A} \quad\;\; \mathrm{A} \\
\underline{\textsc{m} \longrightarrow m} & \hat{m} & \underline{\mathrm{A}} \quad\;\; \underline{\mathrm{I}} \\
\textsc{m} \longrightarrow \mathrm{M} & \overset{\wedge}{\textsc{m}} & \mathrm{A}\ (\textsc{i})\ \mathrm{I}
\end{array}
$$

$$
\begin{array}{lcc}
m \text{ —— } \mathrm{M} & \qquad\quad \mathrm{M} & \mathrm{E} \quad\;\; \mathrm{E} \\
 & \;\;\% & \\
\underline{\textsc{m} \longrightarrow m} & m & \underline{\mathrm{A}} \quad\;\; \underline{\mathrm{I}} \\
\textsc{m} \text{ —— } \mathrm{M} & \overset{\wedge}{\textsc{m}} & \mathrm{E}\ (\mathrm{o})\ \mathrm{O}
\end{array}
$$

SECOND FIGURE

There are two cases, one in which the major term is a species of the middle term, and the minor term (wholly or in part) is not; the other in which the major term is not a species of the middle term, and the minor term (wholly or in part) is.

$$
\begin{array}{lcc}
\mathrm{M} \longrightarrow m & m \qquad\quad & \mathrm{A} \quad\;\; \mathrm{A} \\
\underline{\textsc{m} \text{ —— } m} & \overset{\wedge}{\mathrm{M}} \qquad m & \underline{\mathrm{E}} \quad\;\; \underline{\mathrm{O}} \\
 & \qquad\;\;\% & \\
\textsc{m} \text{ —— } \mathrm{M} & \qquad \textsc{m} & \mathrm{E}\ (\mathrm{o})\ \mathrm{O}
\end{array}
$$

$$
\begin{array}{lcc}
\mathrm{M} \text{ —— } m & \qquad m & \mathrm{E} \quad\;\; \mathrm{E} \\
 & \;\;\% & \\
\underline{\textsc{m} \longrightarrow m} & \mathrm{M} \qquad m & \underline{\mathrm{A}} \quad\;\; \underline{\mathrm{I}} \\
\textsc{m} \text{ —— } \mathrm{M} & \qquad\quad \overset{\wedge}{\textsc{m}} & \mathrm{E}\ (\mathrm{o})\ \mathrm{O}
\end{array}
$$

THIRD FIGURE

There are two cases, one in which the middle term (wholly or in part) is a species of both major and minor term; the other in which the middle term (wholly or in part) is not a species of the major term, but is a species of the minor term.

m —> M	M		A A I
m —> M	∧ m	M	A A I
M —> M		∧ m	I I I

m —— M		M	E E O
	%		
m —> M	m		A I A
M —— M		∧ m	O O O

FOURTH FIGURE

There are three cases, one in which the major term (wholly or in part) is a species of the middle term, and the middle term is a species of the minor term; another in which the major term is a species of the middle term, and the middle term is not a species of the minor term; and a third in which the major term is not a species of the middle term, and the middle term (wholly or in part) is a species of the minor term.

M —> m	M ∧	A	I
m —> M	m ∧	A	A
M —> M	M	I	I

M —> m	M	A
	%	
m —— M	m ∧	E
M —— M	M	E (o)

M —— m	M ∧	E	E
m —> M	m	A	I
	%		
M —— M	M	O	O

128. Real Nature of Syllogistic Thinking.— The analysis of the judgments and also of the syllogistic forms that has just been made reveals the fact that all judgment, as well as all formal union of judgments in the syllogism, aims at getting a better hold upon experience by organizing it. Organization aims at securing unity and system. Our knowledge of any phase of experience is adequate when we have determined it in its place in the entire system of experience. Much of this organization can take place only by referring the momentary phase of experience to which attention is attracted to some organization already fairly well established. Established organizations exist in the fund of concepts, general notions, principles, truths, and laws that have been accurately established by induction, through the mental activity of the human race. They are presented to each person in the formulas that are given through the various educational agencies, such as the school, the family, the church, etc. They are applied to new cases that need interpretation through the mental function of apperception. Hence we may regard the deductive syllogism as a formal apperceptive process. Concepts, general notions, laws, principles, and other organized knowedge forms may all be regarded as classes or categories that embrace many individuals, cases, or acts. When such categories are brought into including and explanatory relation with special cases and made an explaining medium, they are truly speaking genera or higher classes, and the explained cases are species or lower classes. It is entirely arbitrary and superfluous to limit the terms genus and

species to stereotyped classes that have been given general names. Every act of thinking passes beyond the particular, and, after a firm grasp upon its object as an individual, tends to discover in it the typical and general. Hence every act of thinking is truly a class-making procedure. We may then, without violence to the real situation, construe the judgment and the syllogism, no matter what the figure, as *rational efforts to bring a species into relation with its genus through apperception.* In the simple judgment this is achieved directly; in the syllogism, through the instrumentality of a mediating class. In the case of the negative syllogism a suggested relation of species to genus is repudiated to prepare the way for a more adequate organization in a later reasoning process.

129. Relation between Induction and Deduction.—What is the relation between induction and deduction in the systematization of knowledge? Are they independent functions of the mind, each making its contribution separately to the form which finished experience must assume, or do they act in unison and together? The latter alternative is the correct answer. Induction and deduction are simply different abstract conceptions of one and the same mental activity, the relating work of the mind. They have been treated separately because they can be analyzed apart, and such treatment is a need of exposition, not because they exist apart. That this view is the correct one a moment's consideration will show. Suppose that we imagine the case of an actual organization of experience. Let us take the case of experiment to determine the therapeutic effect of

Roentgen rays. Suppose that the aim was to see how they would affect tumors. A physician experimenting in this direction would use the rays on a case and observe the developments. His observation after the first trial would establish a core of experience with which later experiments would be organized piece by piece. At the same time this first observation would serve as a guide, both in method and in anticipation, of the results of the next experiment. The accumulation of experience piece by piece is induction; the method of attack and anticipation of results is deduction. Yet in fact the two proceed together and not separately. Just as the entire body is involved in every step forward, no matter whether the left or the right leg be swinging into the next stride, so the entire activity of organizing knowledge is taking place no matter whether the facts revealed by the experiment be thought of mainly as items in a series of such facts or as grounds for method and anticipation of coming facts. It is largely a matter of attention after all. Let the focus of consciousness be upon the fact as needing adjustment to other facts of like nature, and we give to the process the name induction. But let the results of organizing our experience be regarded as a means of recognizing or anticipating other experience, and the name accorded to the process is deduction. But, as a matter of mental operation, the two functions go on *pari passu,* just as a man does not walk now by hopping a while on one leg and then a while on the other, but by using both in rhythmical forward movement.

So it is with every other instance of accumulating and

organizing experience. It is as true of the unconscious growth into expertness in one's vocation as in the more artificial and conscious illustrative case before us. Go anywhere seeking employment, and the first question asked is, What is your experience? The meaning of this question is always double. What facts of experience have you had an opportunity for accumulating? is the question in its inductive aspect. Have those facts fallen upon fertile soil—are they so related that you can make use of them in application to new problems of the same general nature? is the question in its deductive aspect. But the mind that has the experience, the person who is put to the test by the question, is not a bit of duplicate mental machinery, working now awhile in the one way and then awhile in the other. He both *has* the facts and *knows how to use them,* when questioned, or he must give place to some one who is experienced.

We have spoken of organization and system. What do the terms mean? Does organization come by attaching fact externally to fact? Is it a mere mechanical increase in bulk like the bigness that a snowball or stone acquires? Is it only necessary to let a mind loose in a world of facts on the assumption that they will cluster together and cling mechanically and from the outside like the bees in a swarm? Such views of experience have occasionally prevailed among philosophers in the past. But they cannot maintain their ground in face of the most fertile scientific concept of our age, that of evolution. No, experience is not organized in these mechanical ways. It is a growth and reorganization from within, just as the body is due to an accumulation

and reorganization of materials that are foreign to its nature until broken down and built up anew in other combinations. Knowledge grows in such a way that all past accumulations of fact are modified by the new fact, while at the same time the new fact is assimilated to the old stock of ideas. Induction stresses the modification superinduced upon the old fund of partially organized knowledge by the new fact, deduction the assimilation of the new fact by and to the old. *Growth* can take place only when the old is modified by the new and when the new is modified by the old. Senility shows itself in the failure of the decaying mind to receive the vitalizing touch of the new; rashness and immaturity in the failure of the child-mind to furnish a stock of experience adequate to the task of assimilating the new. True progress comes somewhere between infancy and the involution of old age.

But if this be the true conception of experience under the influence of inductive-deductive organization, it may be asked whether knowledge in an ultimate and satisfactory form is attainable? To ask this question is to answer it in the negative. If knowledge be a growing process, there can be no ultimate end to it any more than there can be to the phenomena of growth in the material world. The individual as such may stop growing in experience, just as the tree may decay and fall; but the tree leaves its offspring in the saplings that are to take its place when it has crumbled into fertilizing dust. The process of evolution is carried forward by the young oak after the parent has fallen. Knowledge is further organized by the needs of the rising generation

after the forefathers and their experiences are matters of history. Just as, to quote the poet, "men rise on stepping-stones of their dead selves to higher things," so we rise out of the inadequately organized knowledge of life's one moment and one generation, as expressed in so-called "truths," to more adequately organized experience. It cannot be otherwise if evolution gives the true description of things. The universe itself is undergoing change and modification; it too is growing. Hence knowledge must mean a constant readjustment to varying conditions. Truth, then, in the shape of a changeless stock of organized experience, is not the aim. It would have no more bearing on the situation, even were it possible, than the skeletons in a graveyard have upon the warm and pulsating issues of life. We must give up such notions of truth, and rest content with the sturdy, growing organism which it really is. It is something whose nature must be conceived in terms of *becoming* rather than of *being*. And we know at least this of becoming: it never rests content in any phase of development, but rushes on to something new that utilizes the old but never exactly reproduces it.

130. Relation of Reasoning to Action.—That reasoning takes place in contemplation of action has already been suggested (p. 272). It remains to consider the relation a little more fully. Motor activity is distinguishable into several classes, the most important of which are reflex, instinctive, habitual, and deliberative. Reflex action is illustrated in the heart-beat, knee-jerk, etc. It presumably depends on a thoroughly developed nerve connection and is functional even before birth.

Instinctive action is illustrated in the tendency to take food, to grow angry, to be frightened, etc. It likewise depends on a pre-established nervous organization, but one that is not so mechanically complete as that required for the reflex type. Habitual action is illustrated in sitting and standing attitudes, typical modes of thought, preference for beginning certain series of performances like lacing the shoes with one hand rather than the other, etc. It represents nervous organization that is acquired during life, and hence varies with the individual. It is therefore far more subject to direct control in the formative stage than either of the others just mentioned. Nevertheless, it finally becomes so automatic and mechanical as to render effective control impossible, a fact which leads the neurologist to suppose that the nervous organization upon which it depends becomes fatally complete in the course of time. It is presumable that reflex action began earliest in the process of animal evolution. Hence we might call it vital habit, since it represents for the most part those responses to essential stimuli without which animal life could not be maintained at all. Next in lineage is instinctive action, representing response to stimuli that further or retard the welfare of the race, yet are not so immediately essential to mere individual life as such. We may therefore call the instincts phyletic habits, since they prompt actions whose main end is to maintain the race or kind. Finally, habit in the usual sense is acquired in each individual's life, and hence might be designated as ontogenetic habit. In fact, it is sometimes helpful and sometimes a hindrance to our inter-

ests, though quite possibly it is a hindrance only indirectly by reason of the antagonism of certain habits to others that are more important.

It will be noticed that in all the three kinds of action described, reflex, instinctive, and habitual, the situation to which the response is required is relatively permanent, as the nature of the stimuli discloses. Air is always normally present to stimulate the lungs to the reflex action required of them, and their work must always go on else we die. The blood must constantly keep pouring back into the heart from the veins and go surging out through the arteries; hence here again the stimuli that set agoing the mechanical nervous processes are always changelessly present. And likewise the responsive action must invariably occur or we die. Again, in the case of instinctive action, a relatively steadfast situation presents unvarying stimuli prompting to an undeviating kind of response. The cravings that an empty stomach arouses in the brain are of a permanent type, and they can be satisfied only in one way. The play-instinct is stimulated to function by an environment whose every phase suggests play-responses, and whose every form is an instrument to that end. Likewise in the case of habit we discover reactions to stimuli that are permanent conditions of the individual's life. We form habits of lacing shoes, opening doors, buttoning our coats, playing the piano, etc., because these several lines of activity are permanent responses to phases of environment that change little if at all. The note of all these three great types of action is permanence, invariability, both of stimuli and re-

sponse. The demand upon our nervous system is that it become so far forth a machine for undeviating adjustment to changeless situations. There can be no going wrong just because there is no appreciable element of complexity or variability. Here is no source of bafflement, no element of doubt, no room for hesitation. So far as such actions are concerned our nervous systems are mere wheels in a system of machinery, of which the great steadying force, the fly-wheel, is environing nature.

How otherwise it is with deliberative action! We think when a novel situation requires response. The mechanical types are perfectly inadequate because all is bafflement and doubt. Deliberative action is initial in character. It opens up new beginnings of nervous organization. Under such circumstances an effort is required to bring the situation into comparison with organized experience that will interpret it, and thereby suggest the appropriate adjustment. It is this effort that results in the feelings of fatigue attending reasoning. We may then regard deliberative action as the resultant of motives found among our relatively organized feelings and ideas. The act occurs only after we see that some group of ideas has a definite likeness to the new situation demanding a response. Now logic deals with reasoning processes, hence with deliberative action. It suggests the general method by which experience may be organized and applied so as to make it useful in case of baffling situations. Such situations demand the full adjustment of attention to them and are therefore the mental activities that arouse the completest self-con-

sciousness. Conclusions are reached only when we can find organized experience that bears upon the case demanding response. And no sooner are they reached than they issue in actions or attitudes. In other words, we always deliberate and reason in view of contemplated action; so that reasoning is not an end in itself, but only a preliminary to more accurate adjustment in novel, baffling, and difficult circumstances than we could hope to secure by non-deliberative action. In case the same type of situation recurs again and again after the first reasonings have met it satisfactorily, the whole business is handed over to habit, and reasoning is reserved for other occasions.

A simple instance will serve to illustrate what is here maintained. Suppose we take the case of a young teacher who has to meet a problem of discipline that is a novelty in her personal experience. Immediate response is impossible because of the bafflement which the new situation occasions. Instead of direct response there is a self-examination to determine whether consciousness has a store of organized ideas that bear upon the case. Finding none of a direct nature, she possibly compares the behavior of the child to her own behavior when a child, comparing the data given by her senses in the conduct of the living problem before her with her concept of her own dead-and-gone childish behavior. If no help comes from this reasoning she seeks advice from an experienced teacher. Now what phase of this mature teacher's knowledge is going to have a bearing upon the present case? Evidently that organized experience that comes from having encountered and met

similar situations. Here we have a type of reasoning in A A A, first figure. The mature teacher's experience furnishes the major premise, *Stubborn attitudes on the part of pupils can be overcome by suggestion.* The young teacher's difficulty furnishes the minor premise, *Johnny's refusal to sing is a stubborn attitude.* The conclusion follows forthwith, *Johnny's refusal to sing can be overcome by suggestion.* The difficulty is solved in this conclusion, which is no sooner reached than action on the part of the young teacher follows. *Thus it is seen that deliberation, i. e., reasoning, takes place in the presence of baffling situations demanding response, and that the conclusions of trains of reasoning are the triggers that release the springs of action.*

131. Practical and Logical Reasoning, How Related.—How are practical and logical reasoning related? The one seems to be a far cry from the other. The difference is not by any means so great as at first sight appears. Indeed, logical reasoning is just practical reasoning grown wary of the pitfalls that encompass it. Both in the inductive organization of experience and its deductive application we have found that there are many chances of error. Logic tries as far as possible to plant red lights before these pitfalls. It aims at unusual caution and circumspection in thinking so as to avoid the two great dangers of common reasoning, hasty generalization in organizing experience, and hasty application in using it. It cautions the common man to establish his generalizations more after the careful method of the scientist than after the happy-go-lucky manner which he prefers. It advises him to apply

his generalizations, laws, and principles with practical certainty that the cases to which they are brought are instances needing that particular organization of experience. But, as has been implied already, such caution costs in time and labor. It therefore should be reserved only for the most important situations. And let it be admitted that the majority of the situations of life can be properly provided for by instinct and habit, and that when deliberation is needed it often suffices to reach a conclusion based upon probability rather than logical necessity. Life admits of much that is doubtful and unsettled, and should be met on numerous occasions in the spirit of faith and trust rather than that of absolute knowledge. Even our most cherished "truths," as was shown a few pages back, must change somewhat with the mutations wrought by time. On this point Minto speaks an illuminating word:

> *It "may be specified as an error incident to the practice of the Syllogism, that it inclines us to look for necessarily conclusive premises and to deny all weight to anything short of this. Now in ordinary life it is comparatively seldom that such premises can be found. We are obliged to proceed on maxims that are not of universal scope, and which lend only a more or less strong color of probability to cases that can be brought under them. . . . They are not true for all cases, but some of them are true for most or for a good many, and they may be applied with a certain probability though they are not rigidly conclusive. The plain man's danger is that he apply them unthinkingly as universals; the formal logician's danger is that, seeing them to be inapplicable as universals, he dismisses them as being void of all argumentative force."* [1]

[1] Minto, *Logic, Inductive and Deductive*, p. 213.

REVIEW QUESTIONS

1. *To what error does logical analysis and the order of exposition in a text-book lead? To what is this error similar?*
2. *What is to be understood by mechanism? By organism?*
3. *Why does thinking resemble the latter rather than the former?*
4. *How may thinking be defined from the biological stand-point?*
5. *How may logical thinking be defined?*
6. *How is the A judgment used, and for what purpose?*
7. *What is the purpose in using the E judgment?*
8. *How are I and O related respectively to A and E?*
9. *What are the two meanings of I and O?*
10. *With what other judgments are A, E, I, and O, respectively, combinable as premises?*
11. *What kind of judgment does thinking aim at, and why?*
12. *What is meant by mediation?*
13. *From the point of view of the organization of experience, how would you define the syllogism?*
14. *How are the relations of genus and species brought about in each of the several figures?*
15. *Be prepared to give and explain the diagrams, showing the relation of terms as genus and species in the several figures.*
16. *How does the syllogism aid in organizing knowledge?*
17. *How is apperception involved in the use of the syllogism?*
18. *Explain why induction and deduction take place together rather than successively.*
19. *Is experience ever finished in the sense of being complete and giving final truth? Give reasons for your answer.*
20. *Name the main kinds of action.*
21. *Describe and cite instances of reflex action; instinctive action; habitual action.*
22. *Under what circumstances does each take place?*
23. *To what kind of environment do they respond?*
24. *Contrast deliberative action with them in respect to the situation occasioning it and the type of response.*
25. *Compare logical and practical reasoning.*

EXERCISES ON CHAPTER XIX

1. Name two subjects in the elementary-school curriculum in which the pupil might be liable to the error of hypostasizing analytical abstractions, and show how it could be corrected.

2. Invent another analogy besides that given at the bottom of page 271 that will illustrate the true nature of logical thinking.

3. In the case of two A judgments of your own making show how the relation of species to genus is established.

4. Make two E judgments and show how they differentiate species under a genus. Name each species so differentiated and the inclusive genus.

5. Make an I and an O judgment and prove that they are really subordinate forms of A and E.

6. Show that I and O, with the meaning "*some only,*" are used to create temporary classes, and restate them in the forms of A and E.

7. Construct one syllogism illustrating each function of each figure and show how it conforms to the diagrams representing the several functions.

8. Taking the syllogisms constructed in answer to Exercise 7, show the evidences of an apperceptive activity of the mind (association by similarity) in organizing experience.

9. Make an illustration of your own to show how induction and deduction work together in developing experience.

10. Cite two or three instances from your own knowledge-getting experience to show that knowledge is a growing process involving change in the conception of what is true.

11. Mention two cases where you have reasoned out a conlusion that was soon carried out into action.

12. Be prepared with proof that syllogistic reasoning occurs in practical life mainly in view of a baffling situation.

13. Be prepared to show by a concrete case how advice-getting is in general the seeking of organized experience to interpret a baffling special case.

14. Mention two or three cases in your recent experience where deliberative thinking was demanded to secure satisfactory adjustment to a situation but where probability rather than certainty was relied upon.

CHAPTER XX.—LOGIC AND EDUCATION

THE discussion of the various topics in the preceding pages has suggested certain applications of logic to teaching. It is necessary to consider this subject more systematically in this final chapter.

132. ANAYLSIS AND SYNTHESIS.—Thinking involves analysis of a fact, the determination of its relation to other facts and synthesis with such facts by classification. The purpose of the complex process is to get better control over the situation to which one must respond. The teacher is continually facing facts of the greatest complexity. His response to them depends in the main upon his power of analyzing them into their simpler factors, and then making valid syntheses into classes. This ability to "pick out and put together" is a growth just as is every other mental function. The instinct to analyze and synthesize manifests itself very early; perhaps, indeed, from the first moment of life. But the instinct will grow in the right direction only provided ample situations are presented calling it into action. The subjects of the curriculum stimulate the child to analysis and synthesis. Geography requires that the home surroundings of the student be broken up into the unfamiliar elements of river-valley, mountain-formation, water-shed, etc. Then in turn these factors produced by analysis must be synthesized into classes

in order to develop the abstract concept of these land and water forms. And so it is with the other subjects. Thus it comes about that the quality of a lesson is directly determinable by the amount and character of the analysis and synthesis accomplished. There comes a time in the career of the pupil when he needs to be made aware of this fundamental form of all learning so that he may initiate it independent of the teacher's guidance. Only in this way can self-reliance be secured. It is especially needful that the prospective teacher have this independent power cultivated, on account of the necessity of breaking complicated situations into their elements and responding to the elements rather than to the whole. Discipline, instruction, the personal relations between teacher and pupil, all of them depend upon the skill with which a complex fact is resolved into its elements and the appropriate response to a partial aspect is made.

There is no discipline superior to logic as a preparation for this rigorous type of analysis and synthesis. Logic makes fine distinctions where the ordinary mind sees only unanalyzable wholes. The training which it gives produces habits of analysis and synthesis and likewise makes the student aware of correct methods.

133. Reflective Thinking.—Analogous to this is the power of reflective thinking, more popularly named "abstract thinking." In spite of the prevalent opinion that teaching should employ object-lessons and concrete aids in all cases, the fact seems to be that ability to think without such aids is the final test of mental growth. Pestalozzi, the Swiss reformer, did more than

any one else to base instruction upon "objects" and to bring about the fashion of "objective-teaching" now so common in the elementary schools. But warm as was his advocacy of the use of the senses as a beginning, he equally emphasized the need of developing the abstract general notion as the goal of the educative process. His followers are too ready to forget the latter part of his theory. This tendency is further fostered by the prevalent notion that all learning should result in "pleasurable activity," for which Spencer is especially responsible. It is comparatively easy to teach the concrete and objective, and to awaken a pleasurable excitement about it. Children love movement and change, and one can entertain them successfully by keeping before them objects that give strong and changing sense-stimuli. But it does not by any means follow that such teaching is educative. Indeed, it rarely is educative unless the more important and difficult work of developing the abstract and general from the concrete is resolutely undertaken and successfully accomplished.

To appreciate the need of the abstract and general, and to understand the processes that produce it, the preparing teacher must study logic. Nothing furnishes a better discipline for this purpose, unless it be metaphysics and the abstract branches of philosophy. These latter are not so available because they require a wider and longer preparation than the average teacher expects to make. Logic, therefore, is the best available means for furnishing this consciousness of the value of abstract, reflective thinking.

The fatal consequence of a neglect of the abstract and conceptual is vividly presented by Bagley:

The fact that attention is attracted and held more successfully by the objective and moving than by the subjective and static not infrequently leads to an over-emphasis of objective-teaching. . . . The matter can be easily overdone. . . . The mind that has learned to lean helplessly upon the objective factor will always be weak and flaccid unless a strenuous effort is made to induce conceptual and subjective processes. Concrete images must always be looked upon as nothing more than necessary but totally subordinate means to a much higher end. Teachers have been so frequently urged to avoid the abstract that they themselves are almost afraid to think in abstract terms. . . . Throughout the history of the race, intellectual progress has ever been away from the sensuous and concrete and toward the ideal and abstract. The education of the child must follow the same line. The pretty pedagogical dogma that education should "begin in the concrete, continue in the concrete, and end in the concrete" is probably, next to "education through play," the most pernicious proposition for which the new schoolcraft must render an accounting.[1]

The teacher is daily confronted by problems whose solution cannot be worked out with the concrete facts before him. Planning lessons, working over programs, and similar undertakings involve considering and combining data that can usually be represented in idea only. Unless the teacher is trained to deal with abstract elements and to work them into combination ideally, he will fail to do proper planning, and his activities will be artisanship rather than art.

134. Logic and Systematized Knowledge.— Closely connected with the topics just mentioned is

[1] Bagley, *The Educative Process*, p. 254 *f.*

the bearing of logical training upon systematic thinking. The great affair of education is to teach not mere facts but knowledge. Knowledge, however, depends quite as much upon the relations that our minds make between facts as upon the facts themselves. Nothing is more meaningless than a mere fact; no mind is more poorly trained than one which is crowded with items of information with no inner relations between them. Logic not only gives the methods and forms requisite for the development of true relations; it also establishes habits of relating in a rational way. It insists upon supplanting mere mechanical association between ideas by associations and syntheses that show profound and far-reaching relationships. The classes suggested by logic depend upon attributes that are fundamental and permanent rather than superficial and temporary. Cautions against hasty and ill-assorted groupings of phenomena are continually urged. Besides the making of classes, we are taught to relate classes into larger systems that will conserve the true co-ordinations and subordinations of experience. Thus logic teaches the teacher to build up for himself an exact and adequate system of knowledge that shall stand in his mind as a picture of a rational universe. If the teacher fails in this he is, just in proportion to the seriousness of his failure, incapable of giving to his pupils an accurate body of knowledge. The habit of order is fixed upon the student of logic so firmly that, out of the choas of mere facts, he will develop the cosmos of a rational universe both for himself and for the child who is at a later period to be in his charge.

135. Logic and Method.—The teacher whose training develops habits of systematic and organic thought will seek to present his lessons in a methodical way. Here again logic is of the utmost assistance. Method is necessarily general in so far as it prepares a form of development that is adapted to various kinds of subject-matter. Logic, the science of the form of orderly thought, is the main reliance, though psychology must be appealed to to give the laws of mental development which determine the method. The main steps in method arc: first, getting the facts; second, analyzing them for fundamental attributes; third, organizing them into classes; fourth, formulating principles, laws, and other kinds of uniformity in unambiguous language; fifth, arranging classes in larger systems that express true relations; sixth, applying this carefully formulated and adequately related knowledge to the acquisition of further experience by sane responses to new situations. Though the several mental processes involved are subject to psychological interpretation, it is to logic that one must resort when he wishes to make sure that the processes occur in the proper order and observe due caution.

It therefore is apparent upon a moment's reflection that logical arrangement of the subject-matter of the lesson is quite as important as psychological arrangement, though the two arrangements are more frequently harmonious than conflicting. Poor logical order is not infrequently poor psychological order. In some respects logical order is more important than psychological. The processes involved in the mental act of getting knowledge may be the first consideration when

a new lesson is given, but the logical relations of the lesson are of more consequence when one looks upon the topic as a closed incident in an organic whole of knowledge. Hence, if there is any real difference between the two orders, precedence should be given to the psychological arrangement as a preliminary step, but to the logical arrangement as the final outcome. But what, exactly, is meant by the psychological and logical orders? Confining ourselves to lessons, the psychological order is the sequence of mental activities by which the mind most easily and naturally apprehends a new topic, the logical order is the organic relationships within the topic and between the topic and others. But psychological order secures ease of learning; the logical order gives meaning. Hence there should be a careful restatement of the lesson in logical order if the psychological order has involved any important departure from it. This can often be secured in reviews, which should always aim at a more significant organization of the facts. In most cases, however, the psychological and logical presentations are identical, the steps that develop the topic organically being the very ones which make it easier to grasp. Here method, which makes the lesson easier to learn, also gives it the fullest meaning.

In such a case logic contributes to pedagogical theory its two related methods of organizing subject-matter, classification and division. We have seen how classification aims to establish the wider relationships between facts by putting them into classes that recognize some chosen characteristic while ignoring differences.

It is the method by which relations are discovered. It is the way in which the manifold particulars of experience are comprehended under the category of unity. Without classification facts would have no meaning, because there would be no suggestion from one to another. We should live as the brute does, our nervous life (it should hardly be called mental) composed of episodes that are never united into a drama. We should be the recipients of innumerable disconnected nervous shocks, each a transient factor in our being, each stimulating its reaction, but utterly devoid of anything like what the human mind understands by meaning. That we do not conduct our mental life on this low plane results from the power of the human mind to record and connect. Memory does the recording, classification the connecting.

The teacher can do no less than to accept this as a fact of human mentality and to conform his teaching to it. He must lead his pupils to arrange their facts according to some wise scheme of grouping or classifying. Such schemes are for the most part provided by the meanings which past experience has got out of its facts. They are ready to hand in the great classes and categories that comprehend our common types of experience. The world is a world of chairs, tables, plants, land and water forms, etc., already. The teacher has merely to accept such groups and develop in the pupil the power of identifying new and rather novel instances as members of established classes. It is in division that he is more likely to meet his Waterloo. Division is the method of exhibiting relationships of co-ordination,

superordination, and subordination within any established class. It is the mind's insistence that differences, for the time ignored in classifying, be again recognized as constituting the individuality of facts. It is the sane, scientific attitude that recalls the philosophical intellect, sweeping on to ultimate oneness, to the claims of the mere fact with its brute insistence.

Division should be used in school work to exhibit the relations between classes. By presenting a scheme of division on the blackboard the child can see the connec-

CUSTOMARY PLAN OF LOGIC

Triangles

Equilateral Isosceles Scalene

Use of braces

Triangles —Equilateral
—Isosceles
—Scalene

Use of numerals

I. Triangles
   1. Equilateral
   2. Isosceles
   3. Scalene

tion of classes in a system. Logical division can be arranged either according to the diagram given on page 48, by the use of braces, or by Roman and Arabic numerals and letters. Thus we have three systems of presentation, each with its special fitness for certain kinds of work.

On the theory that the pupil should learn to do by doing it would be an excellent drill for him to make schemes of division after a topic is completed in the various branches of study. Thus parts of speech could be divided into nouns, verbs, etc.; forms of land into continents, islands, peninsulas, etc.; political divisions into European, American, etc.; and historical events into those belonging to ancient history, mediæval history, and modern history. Countless divisions will suggest themselves to the mind of the teacher. The aim should always be to see by these objective devices the true relationships between classes of facts. While it may be urged in objection to these suggestions that the facts must be presented before they can be related, and that the young child's mind is not mature enough to grasp relations, common experience shows that facts should be presented in such a way that relations are perceived at the same time. And in answer to the last objection, it may be urged that children are quick enough to grasp relations if only the facts are themselves within the range of their comprehension. Indeed, long before reaching the kindergarten age children are found making inquiries that show that relations are their main interest. The three-year-old's whys are a groping for relationships that in our impatience with his insistent questions or in our equally reprehensible tendency to treat his intellectual cravings as sources of adult amusement we are all too prone to treat lightly.

136. Learning as an Inductive Process.—The relation of learning and inductive reasoning is a topic upon which the modern pedagogue loves to dwell.

It is only a few years since almost any text-book that paraded the words "inductive method" was sure to find a ready market. Even to-day, though much of this ardor has cooled, we hear inductive procedure talked about as a plan contributing something of unusual value in the child's education. There can be no controversy about the fact that all the child's knowledge, when viewed as a personal accumulation of experience, is inductive. He passes from moment to moment of life and amasses his experiences bit by bit. This is also true of his lessons. The very word suggests a movement from the particular. His school days chain themselves together link by link. It is also largely true that each topic presented to the pupil comes as an item of experience. The pupil develops his own concepts, general laws, principles, and other abstract formulations of experience. Therefore, learning is to be looked upon as in the main inductive. Where the error of some teachers arises is in conceiving that induction is secured by merely presenting the particular without making certain that the right kind of generalization follows. All lessons in a subject of study should lead somewhere. Each particular fact should be chosen as an example of a law. Hence at all places the teacher should be the guide of the pupil. He knows what general truth should emerge as the result of his teaching and the pupil's learning. He should know what instances to choose to secure this truth with a maximum of clearness and a minimum of labor. Hence the main business of the teacher is to select the typical from the insistent host clamoring for the pupil's attention. What subjects

shall constitute the curriculum in the elementary school, high school, or college is really a problem in the selection of types of human experience that best subserve the three great ends of education, character, efficiency, and culture. Then again, there is need of further choice in teaching a branch of study as laid down in the curriculum. Most subjects extend over a period of years. There should be constant adaptation of the subject-matter to the maturity of the pupil, and at the same time there should be real advance from narrow to wide experience. The teacher must use types to secure this advance to general truths. Hence every lesson should be a well-chosen instance of the general system of knowledge which is the ultimate aim. It is only by the selection of types that true inductive development can be reached. If atypical instances are presented, an adequate general truth will not result, because the concept that the child has will answer to the unusual nature of the cases studied. If the instances are too numerous, the mind will be left in a state of confusion. A few suggestions may be cited as general aids in the selection of the typical.

I. The teacher must himself know the essential characteristics of the class of things whose concept he desires to develop.

II. The instances chosen should have the essential characteristics clearly defined.

III. The instances chosen should have the essential characteristics free (as far as possible) from confusing and unimportant details.

IV. The instances chosen should be sufficiently varied to give an adequate notion of the range of the class.

V. Economy of effort requires that the fewest instances consistent with a rich and full notion should be used.

VI. Tests of the correctness of the notion should be made by oral and written description, or, better still, by having the pupil identify new instances.

137. Learning as a Deductive Process.—The use of deductive method has fallen into disrepute in recent pedagogy because it had been so much in evidence previous to the modern reform movement. Nevertheless, deduction is quite as necessary in learning as induction. It has been shown in the general discussion of the relation between them that the mind acquires and organizes knowledge by both methods. Every mental act may be analyzed so as to show the two processes going on at the same time. If this is so—and no one who really understands the facts of mental life can challenge the assertion—the teacher must depend upon deduction quite as much as induction. It may be at once admitted that the learning mind must accumulate a stock in trade, and this accumulation is heaped up inductively. But while this store is being got together it is also being used. Knowledge is not inductively gathered to be locked up as a miser does his riches, to be brooded over but never spent. It is constantly being used as a means of furtherance in relation to all life's interests. In other words, it is applied just as useful capital is put out into the avenues of trade to earn more. It is in this application of knowledge—somewhat organ-

ized and generalized—that the deductive side of learning comes to the fore. For deduction is nothing but the reference of a fact to the principle that interprets it. It is simply the familiar apperceptive activity always seen in learning. Formal deduction is this old, familiar friend grown cautious and self-conscious with years.

In logic we interest ourselves chiefly in formal processes. These we use as a test to determine if the right principle has been adduced to interpret a given case. We may say, then, that application of principles to facts is always a part of the learning mind's business. This is especially the case in the school, for the school is a short cut to organized experience. Much of its curriculum consists of completely formulated generalizations which the race has aceumulated by the toilsome resource of induction. These generalizations are given to the child in the form of maxims, principles, classes, etc. The teacher who has the "vocation" will see that the fundamental truths are given and properly applied. The application to new cases is deduction. It is a work of supererogation to pass the child through the tedious activities by which these truths have been won out of the big chaos of primitive experience. As well might one argue that the child should be taught to lie and deceive so that he may learn the utility of truth by dearly bought personal experience, as to contend that he should plod through primitive number-processes to realize the more refined principles underlying modern arithmetic. What are ancestors for, anyhow, if not to do their work and transmit it accomplished so as to make our paths easier? Hence it is idle nonsense to withhold from the pupil or-

ganized knowledge-forms. They are to be given as principles to be applied deductively to the illustrative instances that the teacher suggests. They are worthless as mere memorized truths, but are invaluable when made real by application to the cases falling under them. Problems in arithmetic are examples of such cases. The principle is understood when applied to them, and they are understood when brought under the principle. Much the same is true in elementary science. The pupil must take some truths from the teacher's lips and learn what they mean by applying them to the proper cases. To ask him to rework the slowly evolving discoveries that constitute the history of science would be like demanding that the student of ancient history should relive the lives of the Greeks and Romans.

This brief plea for a sane view of the method of discovery should make it evident that a heavy burden of responsibility for early education rests upon deduction. In the simpler phases of teaching, the general idea, the principle, and the rule are applied directly to the case that comes under them. There is then no special difficulty. All that is necessary is to make sure that the right rule is applied. This can usually be accomplished by a very little analysis of the particular case. Such analysis differentiates the essential features from the accidental circumstances, and by means of the former the case is recognized as an instance to which a given rule is applicable. This is the manner of analyzing a problem in arithmetic in order to find what rule will solve it.

But it often happens that a principle can be brought to bear upon a special instance only through the use of a middle term. Under such circumstances we have a situation essentially like the deductive syllogism. There is great danger that the relation of the principle to the middle term, or of the latter to the third term, may not be certainly established; and that no certain relation of the principle to the case that we seek to subsume under it will be established. In such a case the formal type of deductive reasoning is necessary in order to test or verify the relation between the rule and the particular under it. It is when verification is necessary that the teacher needs to call upon his knowledge of the syllogism. Admitting that logic cannot be directly taught to very immature pupils, yet it is clearly possible for the teacher to be conscious of the character of a mediate reasoning process and to direct the pupil's attention to the more obvious forms of error that may arise.

138. Prevention of Fallacy in Learning.—This brings us to one of the most important parts of practical logic—fallacious thinking. The teacher should be on the alert to detect fallacies in the first essays at reasoning. For fallacies, unless noted, pointed out, and avoided, may easily become habitual. Among the most dangerous of the fallacies are hasty generalization, erroneous causal connection, equivocation, accident and its converse, irrelevant conclusion in its several forms, begging the question, and consequent. Children are prone to fall into these errors. They are easily swept off their feet by eagerness to reach a conclusion; they tend to reach sweeping conclusions without a due can-

vass of the facts; they confuse accidental concomitance with cause and effect; they are ready to hold either that rules should have no exceptions or that no rule can be developed from vagrant and ill-assorted particulars; they lose the issue of the argument under the sway of emotional stress; and, in fact, they are an easy prey to all the major forms of error in thinking. We cannot hold them to the rigor that we naturally expect from their more experienced elders, yet at the same time we should not be indifferent to careless reasoning in the schoolroom.

The teacher who is expert in analyzing argument will find many opportunities to lead the young minds under his charge into more accurate ways of thinking. He will make it a point to establish correct habits of reasoning. He will make his pupils conscious of the pitfalls that beset their path, and will thus give them timely hints of warning. By the time the pupils have reached the seventh and eighth grades they can be taught to appreciate clear-cut and logical reasoning, and can by that time be called upon to practise it in simple debates and similar exercises. Throughout the secondary school course much attention should be given to argument of a more formal kind. If this were carefully and conscientiously done, there would be far less of the slipshod argument that passes among the unskilled for wisdom than is at present the case. In a democratic country where constant appeals are directed to the supposed reasoning powers of the average man, it is of the utmost importance that there should be an average ability to detect and reject the grosser forms of error in argument.

Yet this power of logical discrimination is not a common accomplishment, as may be judged from the flagrant and often intentional errors of public speakers and controversial writers. Demagoguery flourishes upon sophisms, and the mob are greedy for the sop that the cunning demagogue throws to them. The schools could do much to develop a sharper logical acumen. And they should do their utmost in this direction, because sanity of action depends so largely upon a keen discrimination between the plain truth and the rhetorical tricks which disguise error until it bears the similitude of truth.

The teacher is the victim of many ill-advised educational plans. Even the educational theories of the great writers upon education exhibit traces of fallacious reasoning. It is necessary that he have practice in detecting fallacies in order to sift the true from the false. The use of the term "nature" by Comenius and Rousseau is so ambiguous that fallacious argument is to be found in many parts of their works. Spencer is guilty of palpable blunders in reasoning. Unless the teacher can detect these fallacies he will accept conclusions from invalid arguments. These conclusions will guide his policy and work havoc in education.

139. Definition.—Definition-making should begin early in the pupil's school career. Moreover, it should be logical. After the facts are classified properly, after the denotation and the connotation are established, after logical division has shown systemic relations, logical definition should render this work permanent as part of the pupil's intellectual resources. In the

making of definitions his own initiative can be put in play. Defining is the acme of clear thinking, and it is almost sure to come easily if the data have been properly classified. The aim should be to define briefly, in the child's own vocabulary, positively, and so as to suggest relation and distinction. This can be successfully done if the teacher will only abandon ready-made definitions and set his pupils to accurate thinking. We have been too much inclined to attach a kind of sanctity to textbook definitions. We allow ourselves to fall into the *argumentum ad vericundiam,* and show all too much reverence for the words of the authors. There is no reason why a child who has made a careful study of islands and has the proper notion of their difference from other land forms should not couch a brief definition in his own simple words. Such self-activity is of inestimable value to intellectual development because it leaves a fine glow of the joy of discovery. It also develops habits of precision in thought and expression.

The making and accurate defining of an educational terminology is a pressing need of the times. If education is to take scientific form, it is especially necessary to have exact and technical language to express it. The present vocabulary of education is largely borrowed. Psychology furnishes it with many terms. Biology is just now contributing many others. Unfortunately, the nomenclature of psychology has come in the main from common speech. Hence educational terms are vague and misleading. This fault may readily be corrected by accurate and painstaking fixation of meaning and definition. Such words as pedagogy, education,

instruction, formal steps, many-sided interest, learning, teaching, knowledge, adjustment, conduct, idea, habit, apperception, self-activity, attention, correlation, moral training, formal discipline, visualization, child-study, environment, and scores of others, would benefit by being less glibly used and more intelligently defined.

These are some of the most evident applications of logic to education. The list is not exhaustive, for logic contributes a general reflective and critical attitude that is of advantage in almost every phase of the teacher's work. Like psychology, it is a general science that should not be expected to solve the trifling details of school life.[1] But it does set a standard that guides the teacher to better thinking on his own part, therefore to better educational leadership.

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Sigwart, *Logic,* Vol. II, Pt. III, Ch. VI.

Venn, *Empirical Logic,* Ch. XXV.

REVIEW QUESTIONS

1. *Explain analysis and synthesis as involved in learning; in teaching.*
2. *Upon what does the worth of a lesson depend, and why?*
3. *Show specifically how logic has trained you in analysis and synthesis.*
4. *What kind of thinking should be secured in the mental growth involved in education? Show why such thinking is desirable.*

[1] On this point *cf.* Welton, *The Logical Bases of Education,* p. 248.

5. *What danger can you see in too much emphasis upon the concrete and particular?*

6. *How does power to do "abstract" thinking help the teacher?*

7. *How is "knowledge" related to "fact"?*

8. *Why should one comprehend the universe of experience as a related whole?*

9. *How does logic contribute to general method?*

10. *What do you understand by the logical as distinguished from the psychological order? Which is more important from the stand-point of the lesson? from the stand-point of the learning process?*

11. *Do the two orders usually conflict or agree? In case of conflict, when should the psychological order be given the preference? When the logical?*

12. *Could there be any knowledge without classification and system? Give reasons for your answer.*

13. *How may logical division be utilized in blackboard work? What is its use from the teacher's point of view? from the pupil's?*

14. *What should be the goal of inductive teaching processes (the so-called "inductive development lesson")?*

15. *What reasons can you assign for the study of the type in nature-study? in arithmetic? in geography?*

16. *Be prepared to discuss and elaborate upon each of the suggestions for the selection of types. Can you think of others?*

17. *When should deductive teaching be used? Why?*

18. *What is the objection to an attempt to use induction to the exclusion of deduction?*

19. *Where does the danger lie in deductive teaching? How may identification of the instance with the general principle be secured?*

20. *Name the fallacies that you would expect children's reasoning to show.*

21. *Suggest means by which the teacher may prevent fallacious reasoning on the part of his pupils.*

22. *In what subjects of the elementary-school course would you especially look for fallacies? of the grammar-school course? of the high-school course? of the training-school course?*

23. *Why does training for citizenship in a democracy require special safeguards against fallacious reasoning? What fallacies does the citizen need to be especially warned against?*

24. *Why is defining important in school work? Suggest means by which you would secure an interest in logical definition among young pupils.*

25. *What is the value of defining to the teacher who is interested in educational problems?*

26. *Discuss and elaborate the proposition that neither logic nor psychology can solve the petty details of the school-room. Do you consider this proposition true, and if so, why? If true, does the proposition imply that they have no value in the teacher's preparation? Give reasons for your view.*

EXERCISES ON CHAPTER XX

1. Cite one instance of analysis and synthesis in the teaching of each of the following subjects (be specific): language, geography, arithmetic, nature-study, history.

2. Name topics in the above subjects that involve abstraction on the part of elementary-school pupils.

3. Indicate topics from the above branches involving relationships between systems of knowledge ("correlation"), such, for example, as the relationship of a fact of history to a fact of geography (be specific).

4. Indicate a lesson in which the teaching should be mainly inductive, and show why. Another in which it should be mainly deductive, and give your reasons. Was the method exclusively inductive or deductive?

5. Select topics from history, nature-study, and geography, and use in each case the scheme of division that you deem most appropriate. Give your reasons for your preference.

6. Select four types from various subjects and show that they conform to the suggestions about the selection of types.

7. Imagine a typical fallacy that a child might make in some lesson, state it as you would imagine that the child would, and show how you would make him see and correct his reasoning.

8. Imagine a lesson in which you are aiming at a verbal statement of a definition, and show how you would lead the child to define correctly.

9. Try to frame a careful logical definition of any five of the terms mentioned on pp. 313 (bottom) and 314.

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